EE 330
Lecture 17

MOSFET Modeling

Prelab Annuncement

A Pre-Lab will be posted on Canvas for Lab 7

Exam Schedule

Exam 2 will be given on Friday March 11

Exam 3 will be given on Friday April 15

Photo courtesy of the director of the National Institute of Health (NIH)
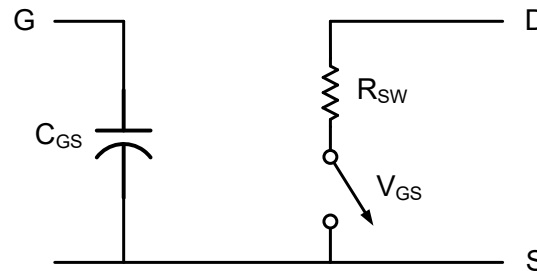
Masks Save Lives

NIH

As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status
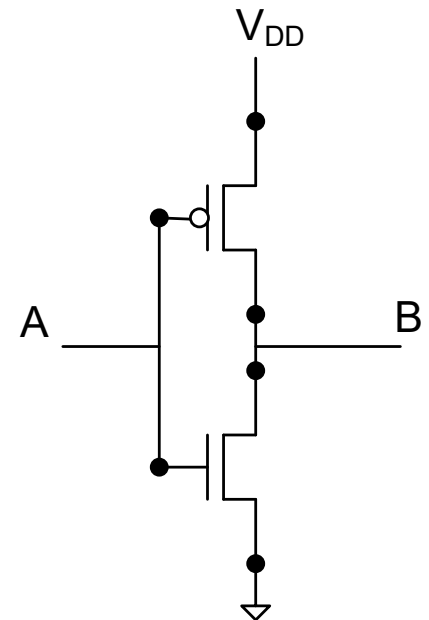
Limitations of Existing MOSFET Models

B

A ———[MOSFET]——— Y

R ⧄
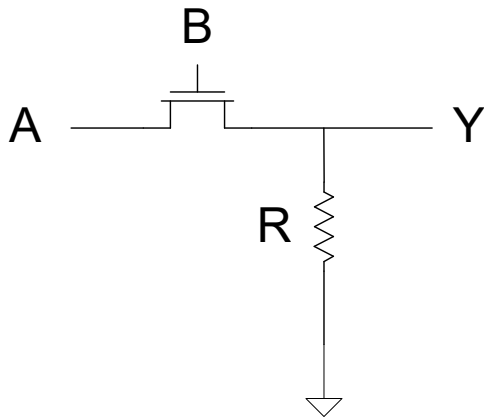
What is Y when A=B=V_{DD}

G ——————— D

C_{GS} ⊣⊢

R_{SW}

V_{GS}

S

For minimum-sized devices in a 0.5u process with V_{DD}=5V

$$\mathbf{C_{GS} \cong 1.5fF}$$

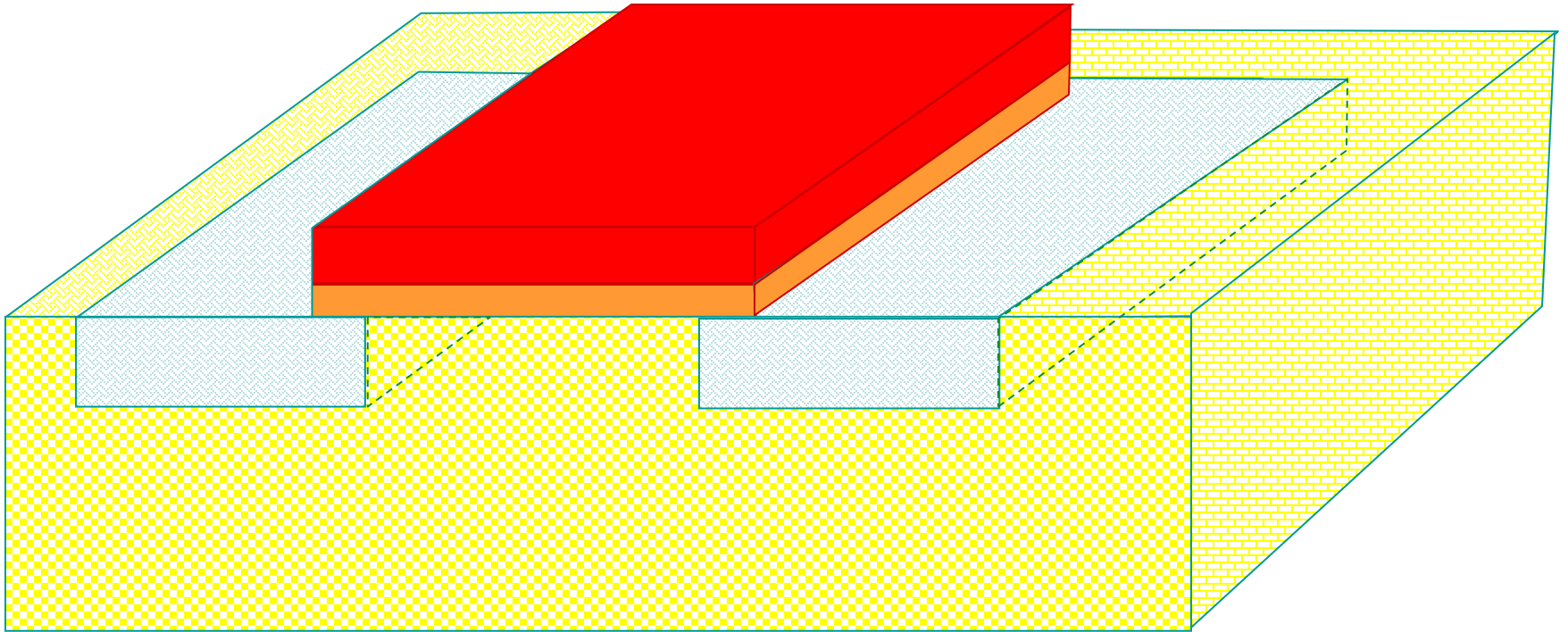$$\mathbf{R_{sw} \cong} \begin{cases} \mathbf{2K\Omega \quad n-channel} \\ \mathbf{6K\Omega \quad p-channel} \end{cases}$$

What is R_{SW} if MOSFET is not minimum sized?

V_{DD}

A ——————— B
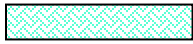
What is power dissipation if A is stuck at an intermediate voltage?

Better Model of MOSFET is Needed!

n-Channel MOSFET

■ Poly	■ Gate oxide	
▨ n-active	▨ p-sub	

n-Channel MOSFET

n-Channel MOSFET

🟥 Poly	🟧 Gate oxide
▦ n-active	▦ p-sub
▦ depletion region (electrically induced)	
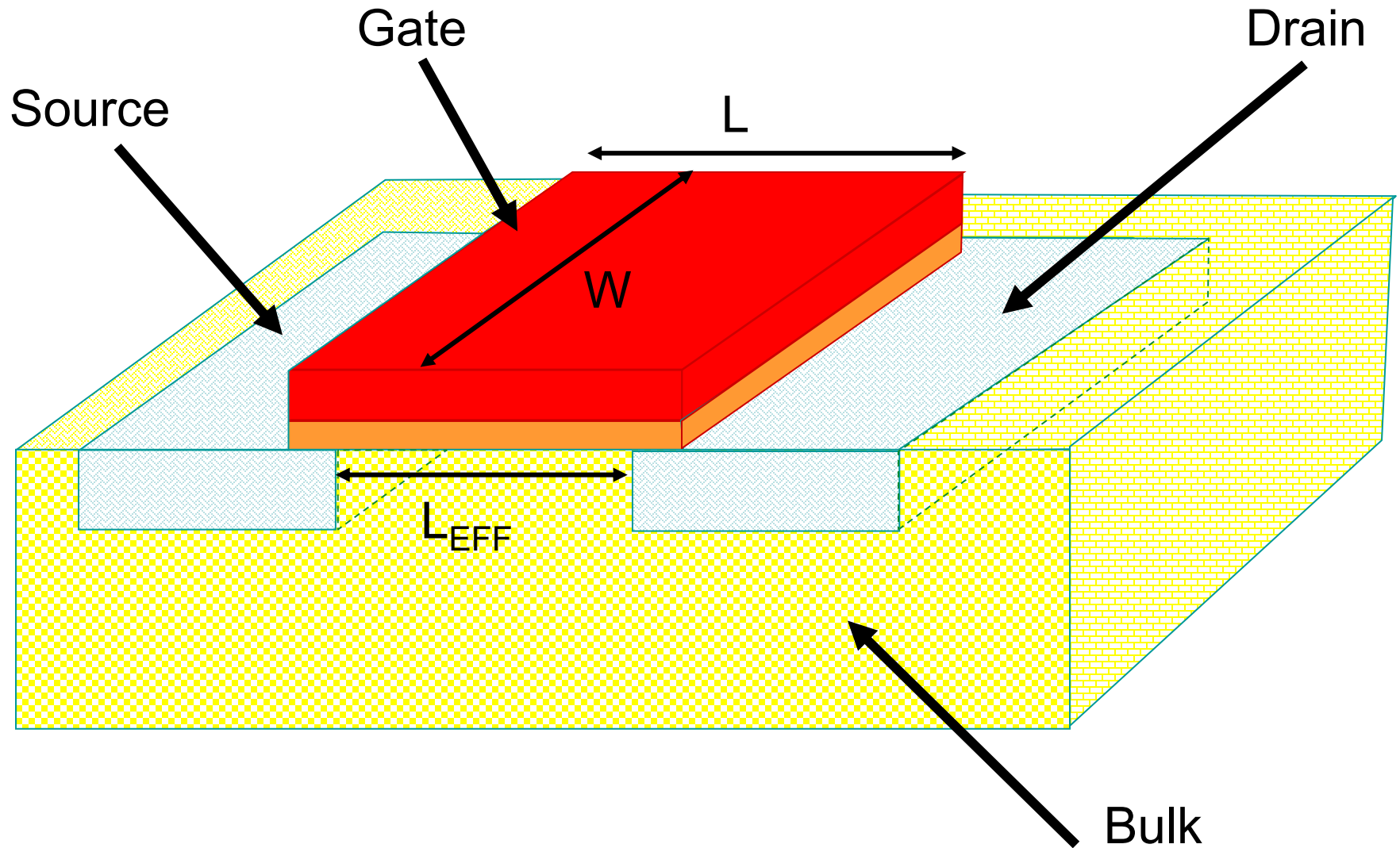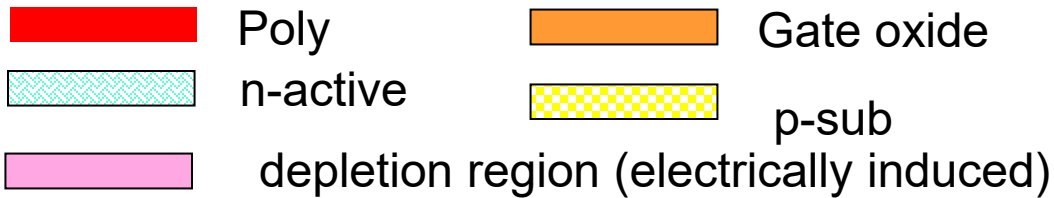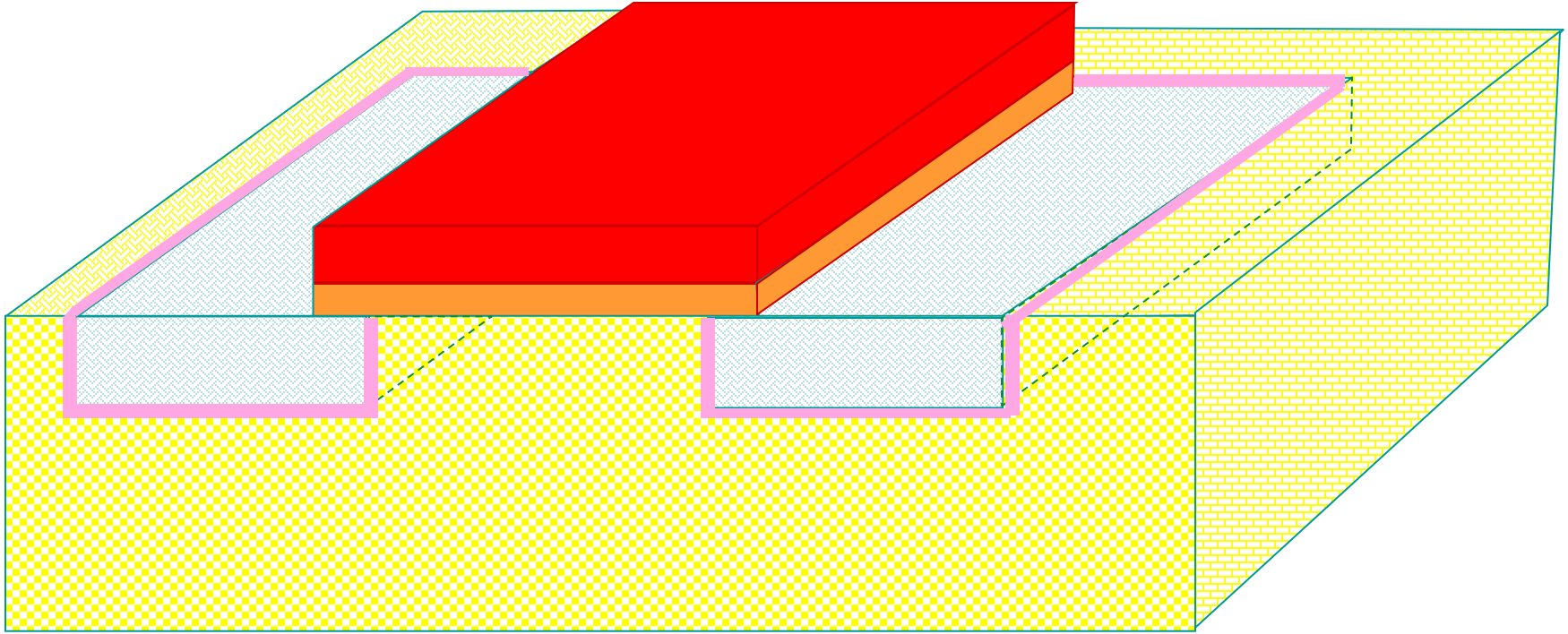
- In what follows assume all pn junctions reverse biased (almost always used this way)
- Extremely small reverse bias pn junction current can be neglected in most applications

n-Channel MOSFET Operation and Model
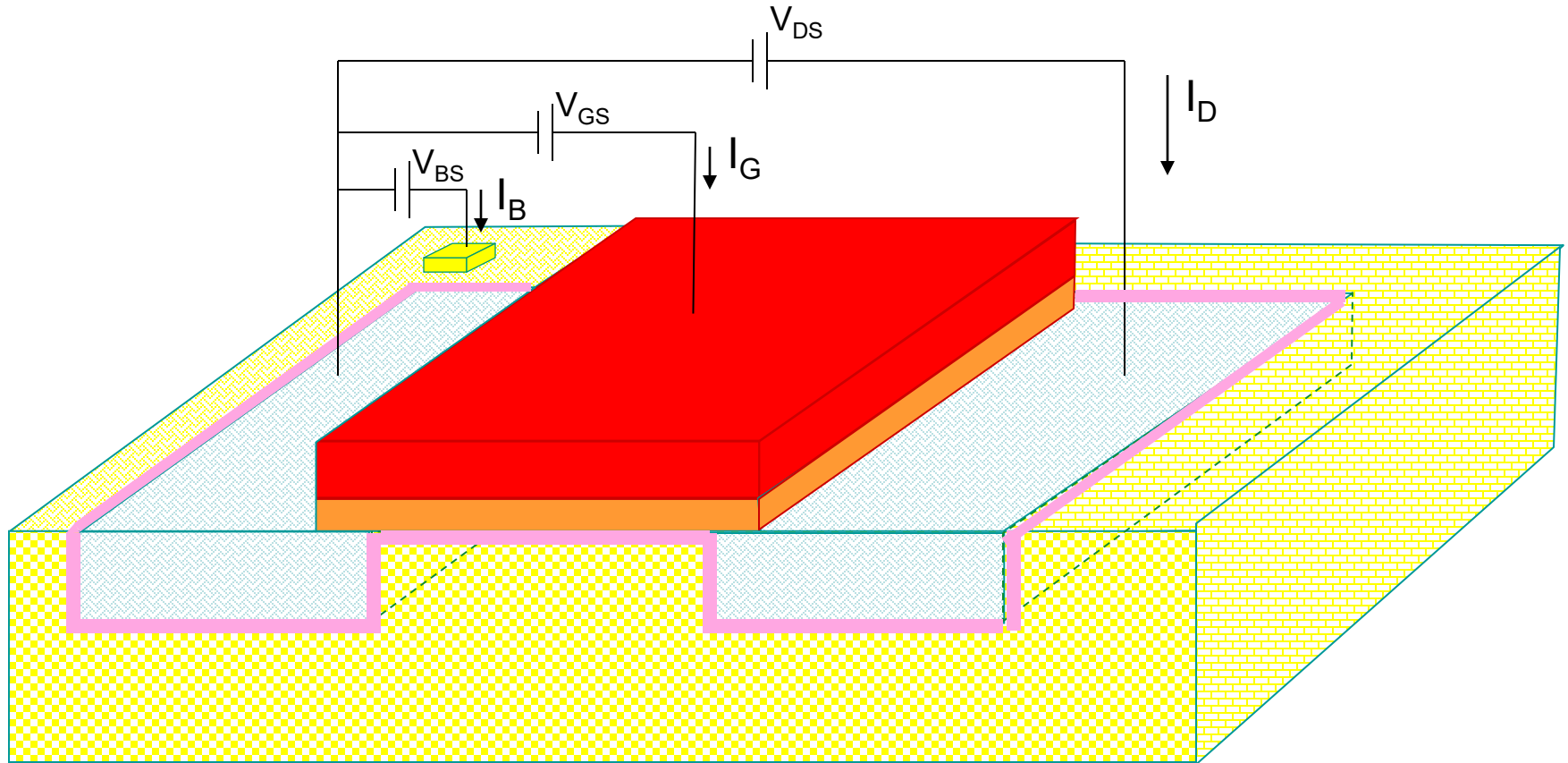
Apply small V_{GS}
(V_{DS} and V_{BS} assumed to be small)
Depletion region electrically induced in channel
Termed "cutoff" region of operation

$I_D = 0$
$I_G = 0$
$I_B = 0$

n-Channel MOSFET Operation and Model
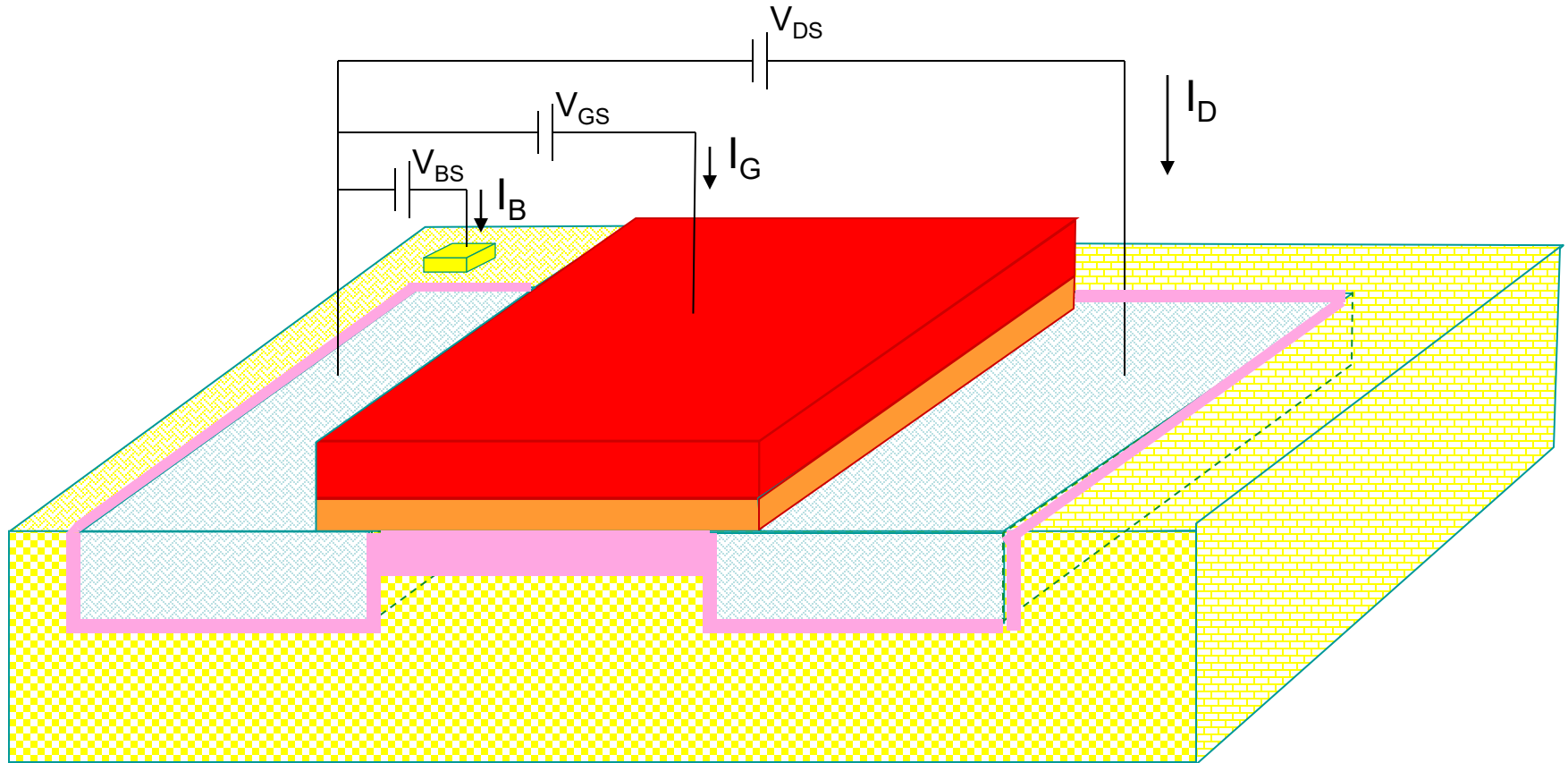
Increase V_{GS}
(V_{DS} and V_{BS} assumed to be small)

Depletion region in channel becomes larger

$I_D = 0$
$I_G = 0$
$I_B = 0$

n-Channel MOSFET Operation and Model

V_{DS}

V_{GS}

V_{BS}

I_B

I_G

I_D
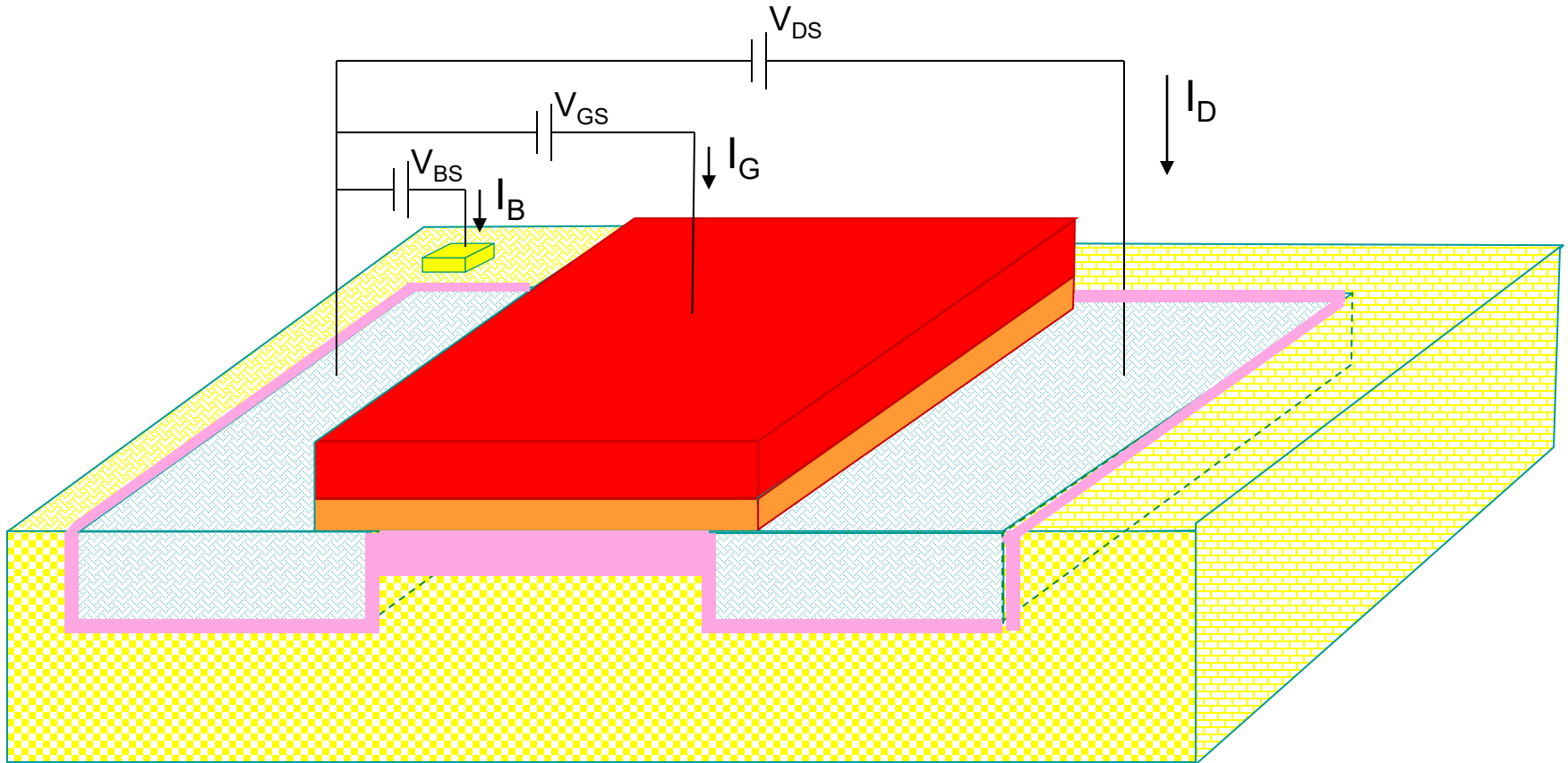
$I_D=0$
$I_G=0$
$I_B=0$

Model in Cutoff Region

n-Channel MOSFET Operation and Model

Critical value of V_{GS} that creates inversion layer termed threshold voltage, V_T)

V_{DS}

V_{GS}

V_{BS}

I_B

I_G

I_D

x

0

L

(V_{DS} and V_{BS} small)
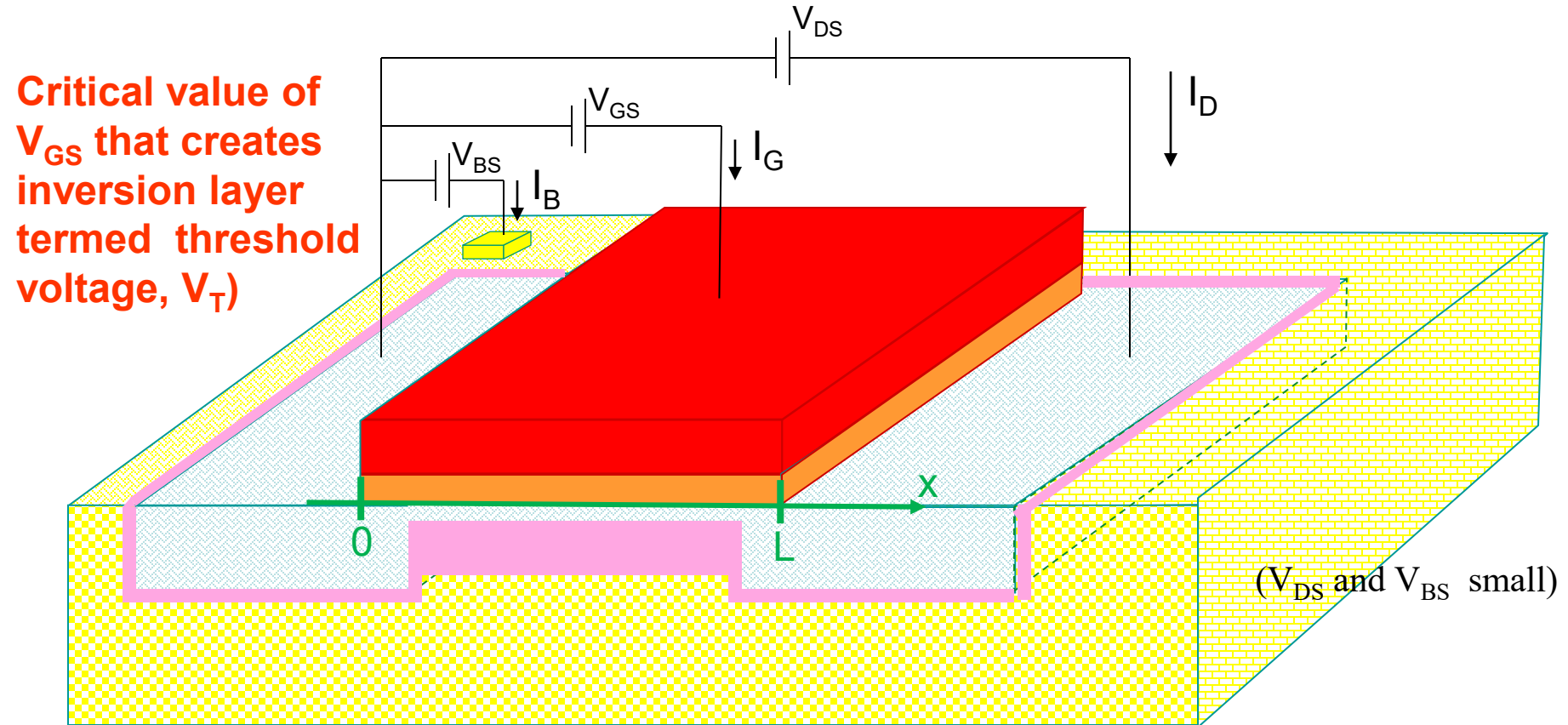
Increase V_{GS} more

Inversion layer forms in channel

Inversion layer will support current flow from D to S

Channel behaves as thin-film resistor

$I_D R_{CH} = V_{DS}$

$I_G = 0$

$I_B = 0$

Triode Region of Operation
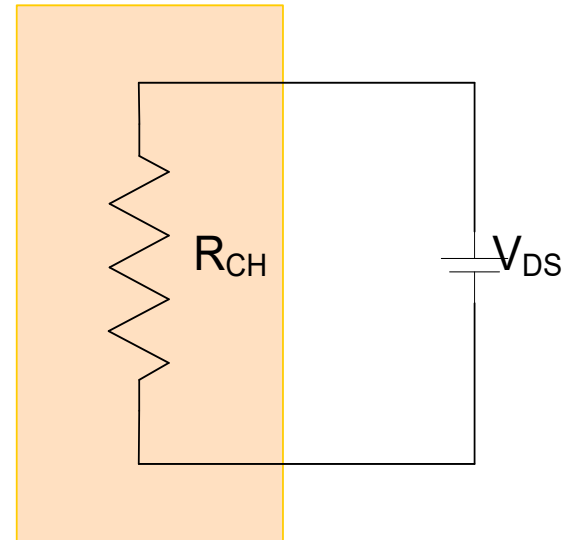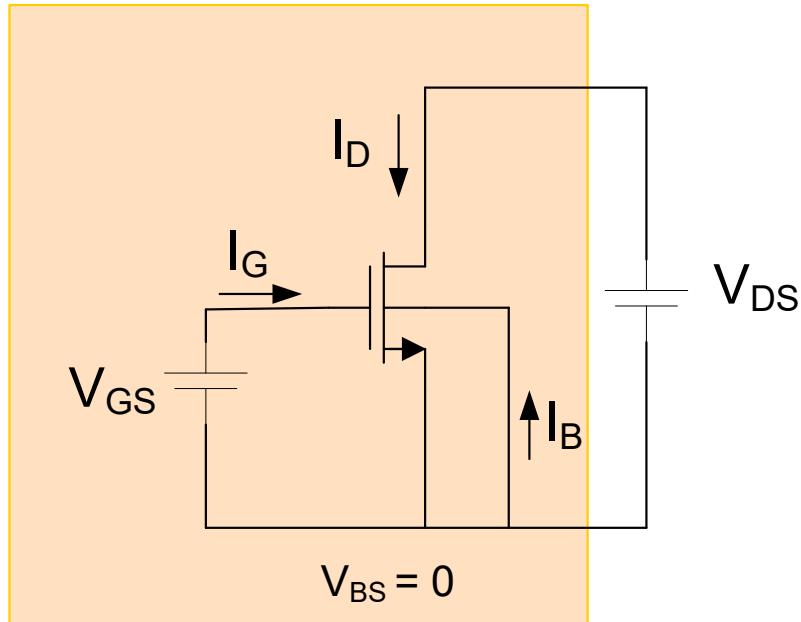
For V_{DS} small

$$R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH}) \mu C_{OX}}$$

$$I_D = \mu C_{OX} \frac{W}{L} (V_{GS} - V_{TH}) V_{DS}$$

$$I_G = I_B = 0$$

Behaves as a resistor between drain and source

Model in Deep Triode Region

Triode Region of Operation
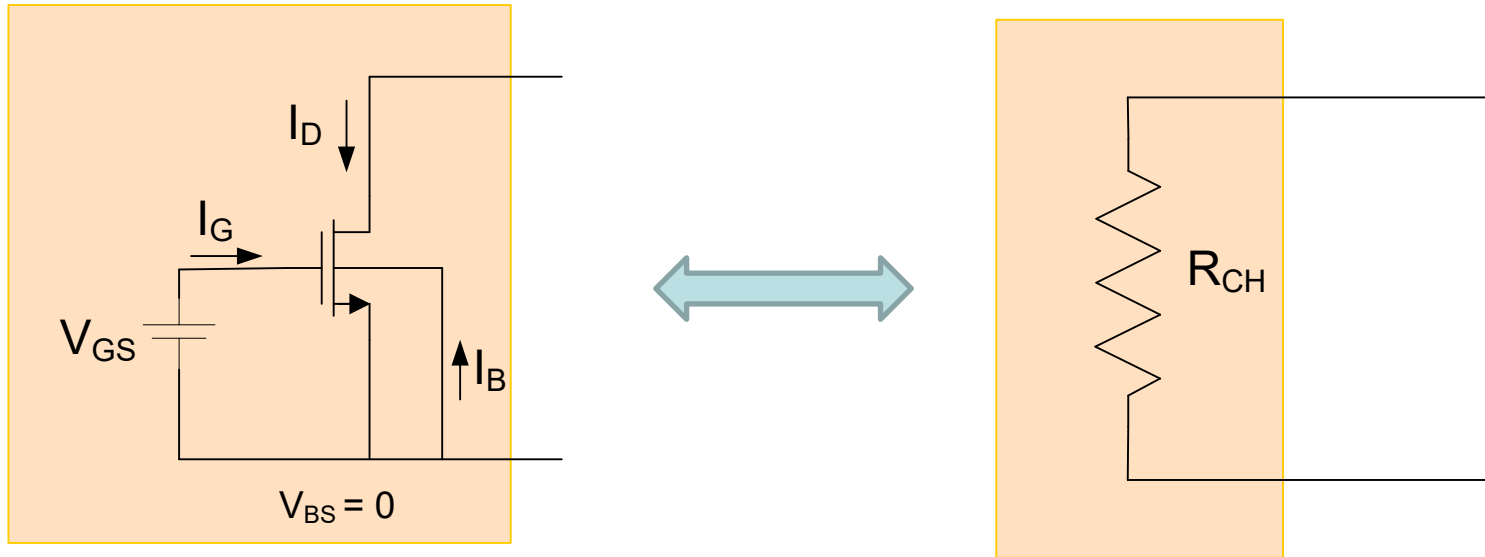
For V_{DS} small

$$R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH})\mu C_{OX}}$$

Resistor is controlled by the voltage V_{GS}
Termed a "Voltage Controlled Resistor" (VCR)

n-Channel MOSFET Operation and Model

V_{DS}

V_{GS}

V_{BS}

I_B

I_G

I_D

x

0

L

(V_{DS} and V_{BS} small)

$V_{GC}(x)$ approx. constant for small V_{DS}
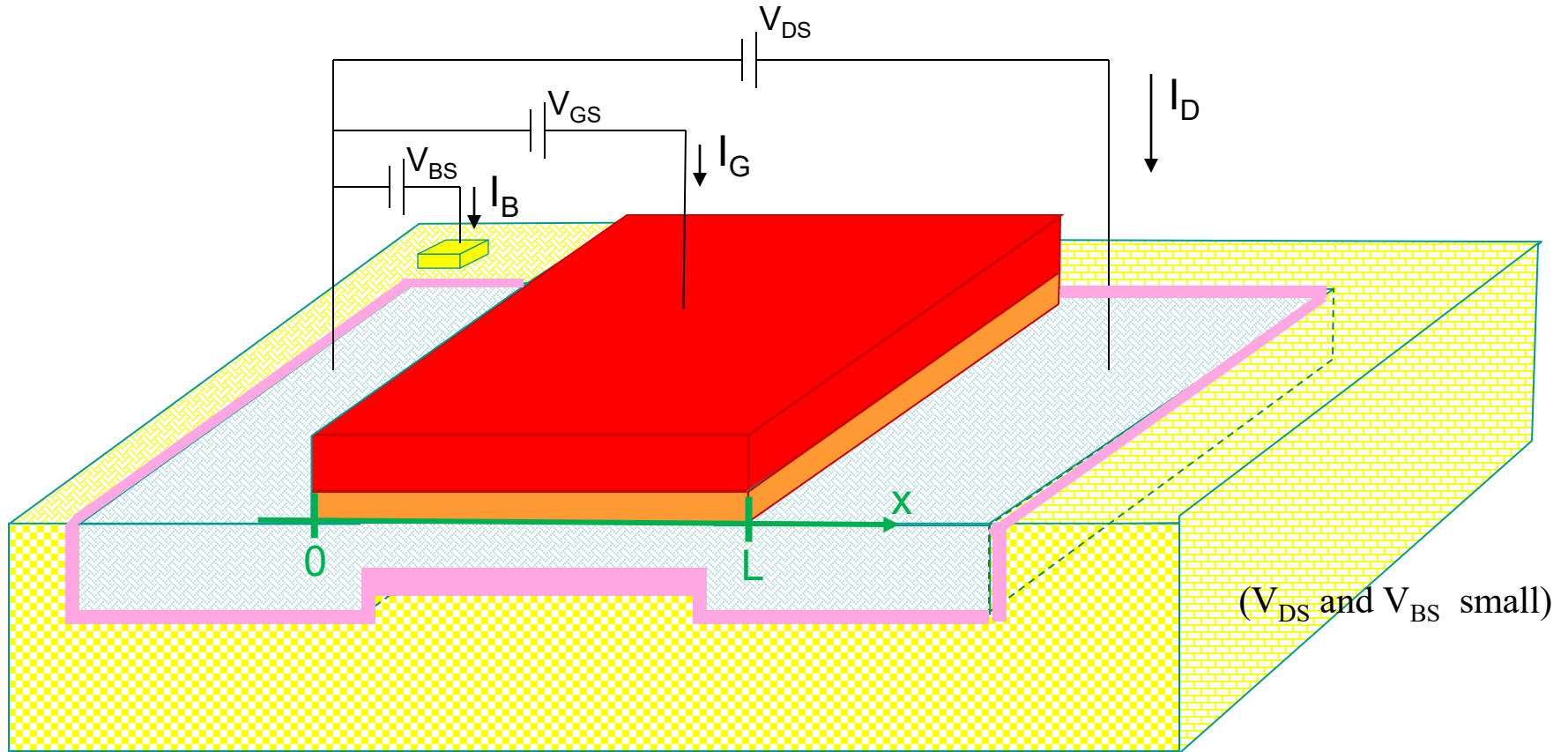
Increase V_{GS} more

Inversion layer in channel thickens

R_{CH} will decrease

Termed "ohmic" or "triode" region of operation

$I_D R_{CH} = V_{DS}$

$I_G = 0$

$I_B = 0$

n-Channel MOSFET Operation and Model
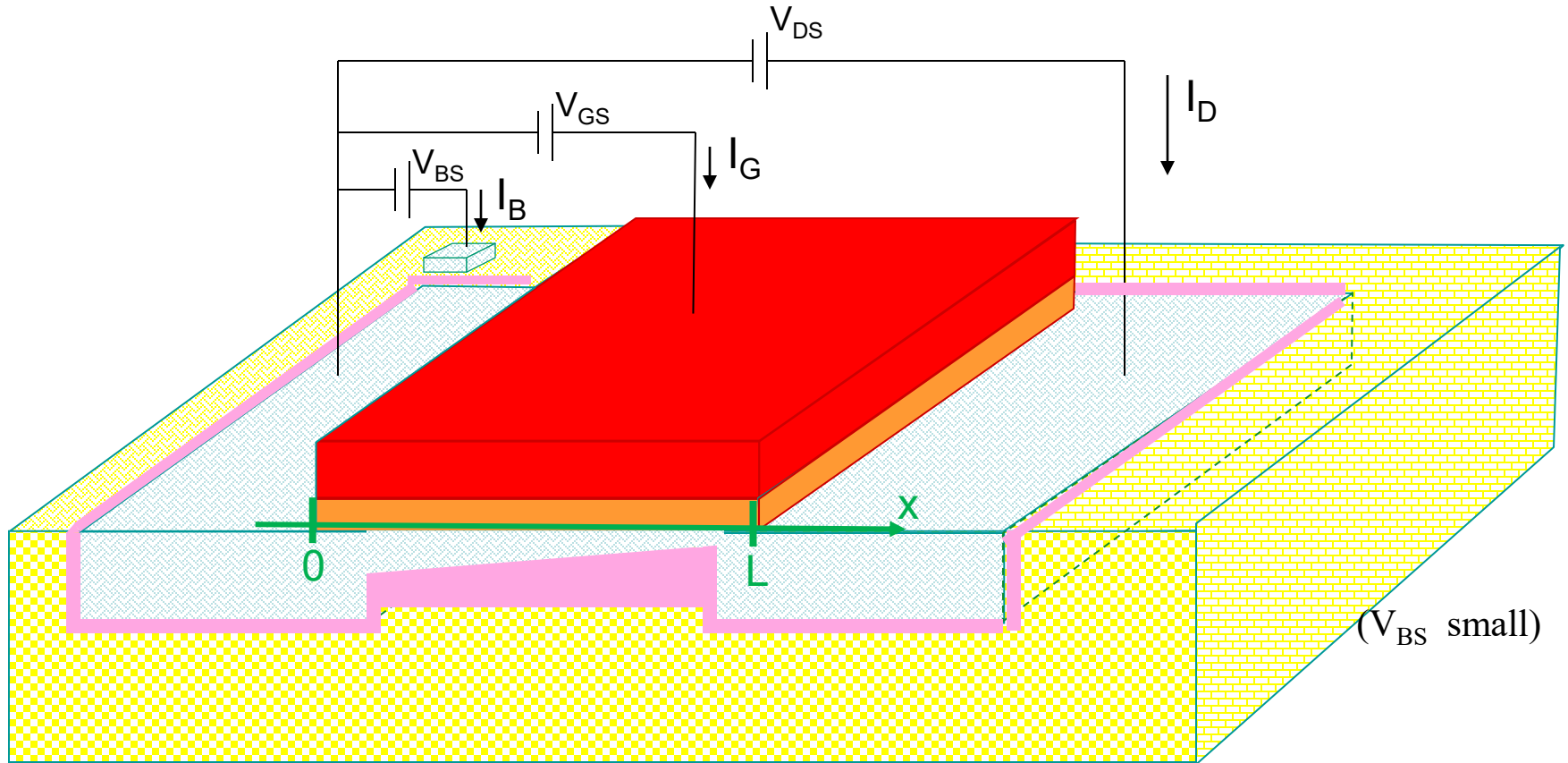
Increase V_{DS}

$V_{GC}(x)$ changes with x for larger V_{DS}

Inversion layer thins near drain

I_D no longer linearly dependent upon V_{DS}

Still termed "ohmic" or "triode" region of operation

I_D=?

I_G=0

I_B=0

Triode Region of Operation
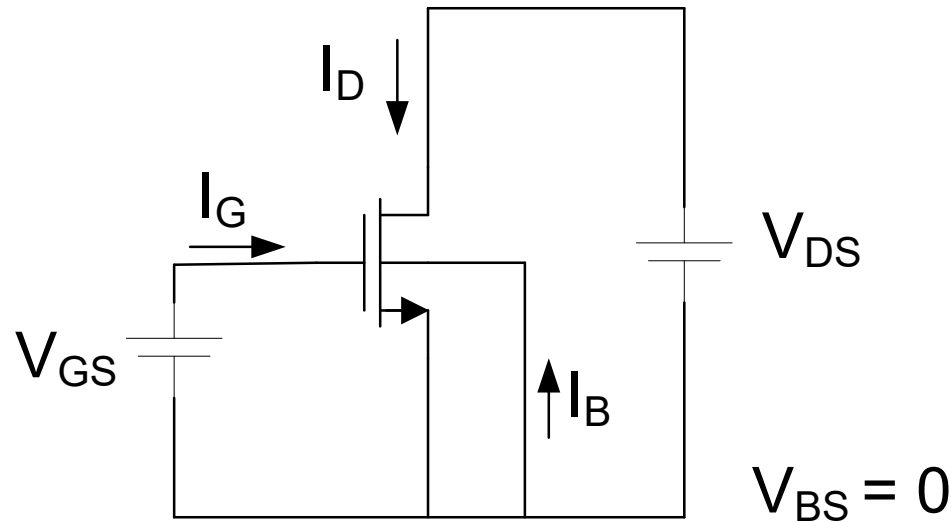
For V_{DS} larger

$$R_{CH} = \frac{L}{W} \frac{1}{(V_{GS} - V_{TH}) \mu C_{OX}}$$

$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$

$$I_G = I_B = 0$$

Model in Triode Region

n-Channel MOSFET Operation and Model
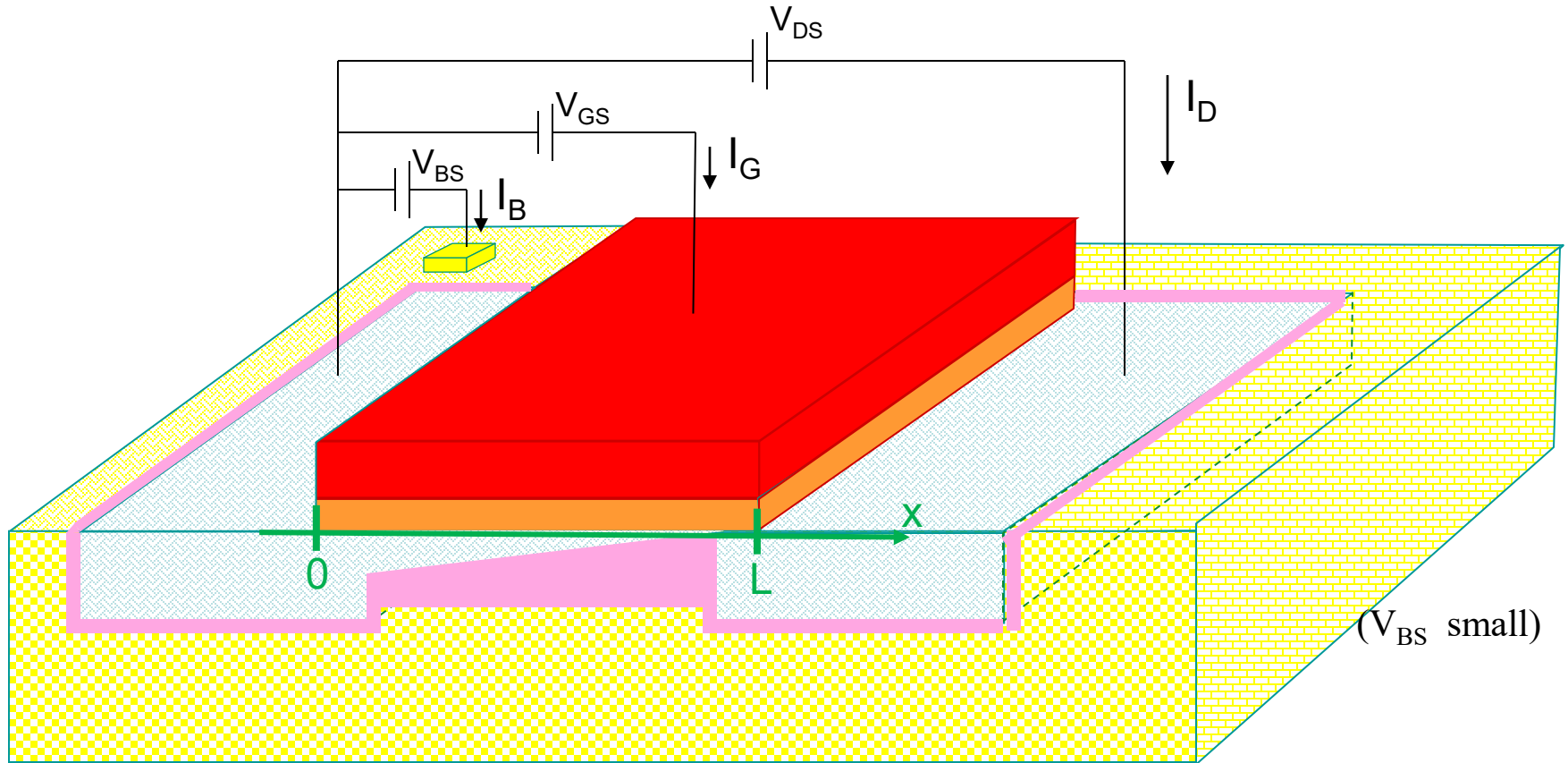
Increase V_{DS} even more

$V_{GC}(L) = V_{TH}$ when channel saturates

Inversion layer disappears near drain

Termed "saturation" region of operation

Saturation first occurs when $V_{DS} = V_{GS} - V_{TH}$

$I_D = ?$

$I_G = 0$

$I_B = 0$

Saturation Region of Operation

$V_{BS} = 0$

$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$
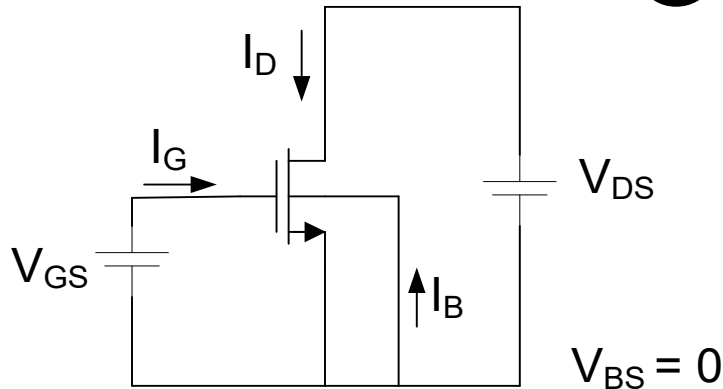
For V_{DS} at onset of saturation ————

or equivalently

$$I_D = \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_{TH} - \frac{V_{GS} - V_{TH}}{2} \right) \left(V_{GS} - V_{TH} \right)$$

or equivalently

$$I_D = \frac{\mu C_{OX} W}{2L} \left(V_{GS} - V_{TH} \right)^2$$

$$I_G = I_B = 0$$

n-Channel MOSFET Operation and Model
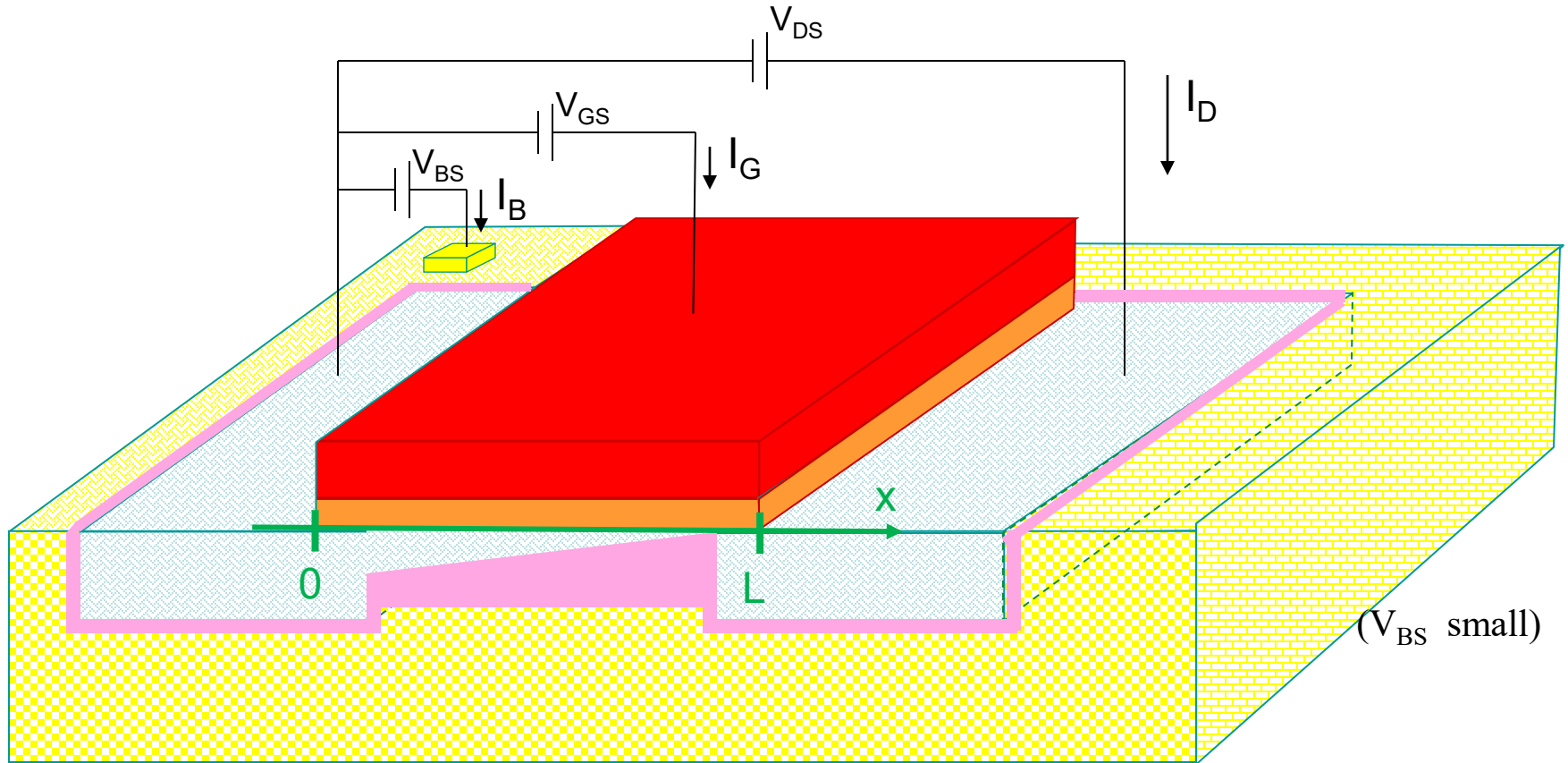
Increase V_{DS} even more (beyond V_{GS}-V_{TH})

Nothing much changes !!

Termed "saturation" region of operation

I_D=?
I_G=0
I_B=0

Saturation Region of Operation
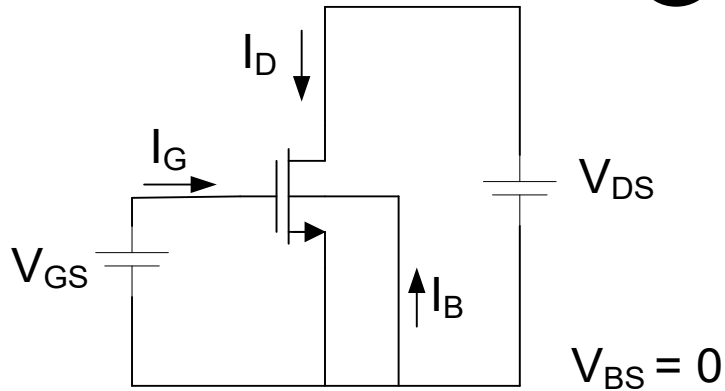
For V_{DS} in Saturation

$$I_D = \frac{\mu C_{OX} W}{2L}\left(V_{GS} - V_{TH}\right)^2$$

$$I_G = I_B = 0$$

Model in Saturation Region

Model Summary

n-channel MOSFET

Notation change: $V_T = V_{TH}$, don't confuse V_T with $V_t = kT/q$

$V_{BS} = 0$
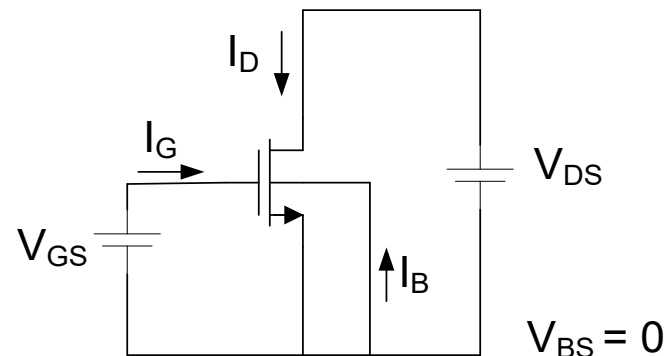
$$I_D = \begin{cases} 0 & V_{GS} \leq V_T & \text{Cutoff} \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T & \text{Triode} \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T & \text{Saturation} \end{cases}$$

$I_G = I_B = 0$

Model Parameters: $\{\mu, V_T, C_{OX}\}$ Design Parameters : $\{W, L\}$

This is a piecewise model (not piecewise linear though)
Piecewise model is continuous at transition between regions

(Deep triode special case of triode where V_{DS} is small $\quad R_{CH} = \dfrac{L}{W}\dfrac{1}{(V_{GS} - V_T)\mu C_{OX}}$)

Note: This is the third model we have introduced for the MOSFET

Model Summary

n-channel MOSFET
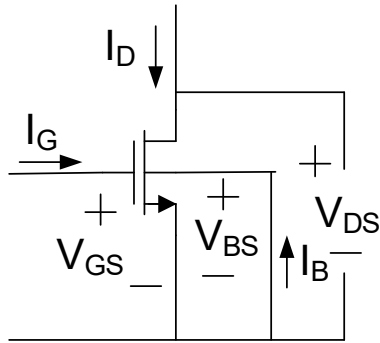
$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$V_{BS} = 0$

$$I_G = I_B = 0$$

Observations about this model (developed for V_{BS}=0):

$$I_D = f_1\left(V_{GS}, V_{DS}\right)$$

$$I_G = f_2\left(V_{GS}, V_{DS}\right)$$

$$I_B = f_3\left(V_{GS}, V_{DS}\right)$$

This is a nonlinear piecewise model characterized by the functions f_1, f_2, and f_3 where we have assumed that the port voltages V_{GS} and V_{DS} are the independent variables and the drain currents are the dependent variables

General Nonlinear Models
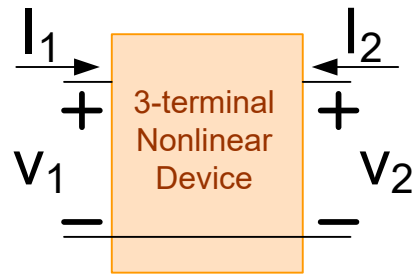
$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

I_1 and I_2 are 3-dimensional relationships which are often difficult to visualize

Two-dimensional representation of 3-dimensional relationships
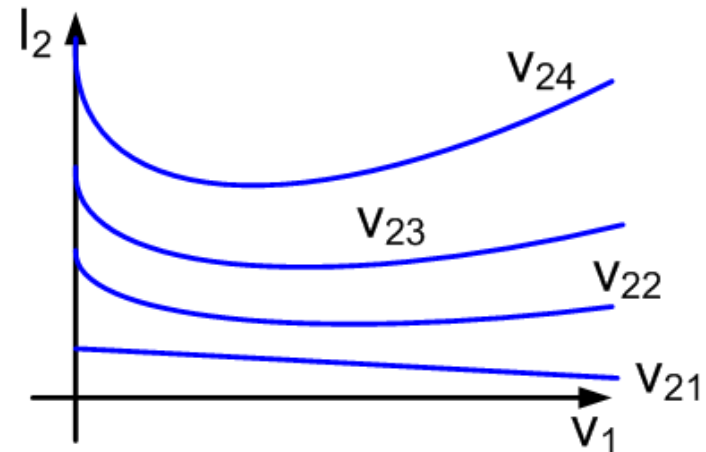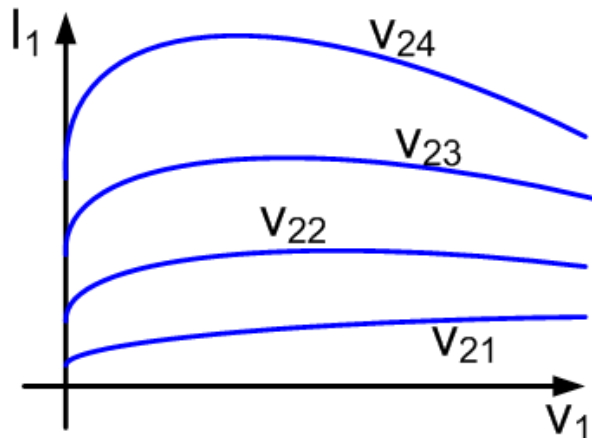
Graphical Representation of MOS Model

$$I_D = \mu C_{OX} \frac{W}{L} \frac{V_{DS}^2}{2}$$

Triode

V_{GS4}

V_{GS3}

Saturation

Deep Triode
RegionRegion

V_{GS2}

V_{GS1}

V_{DS}

Cutoff
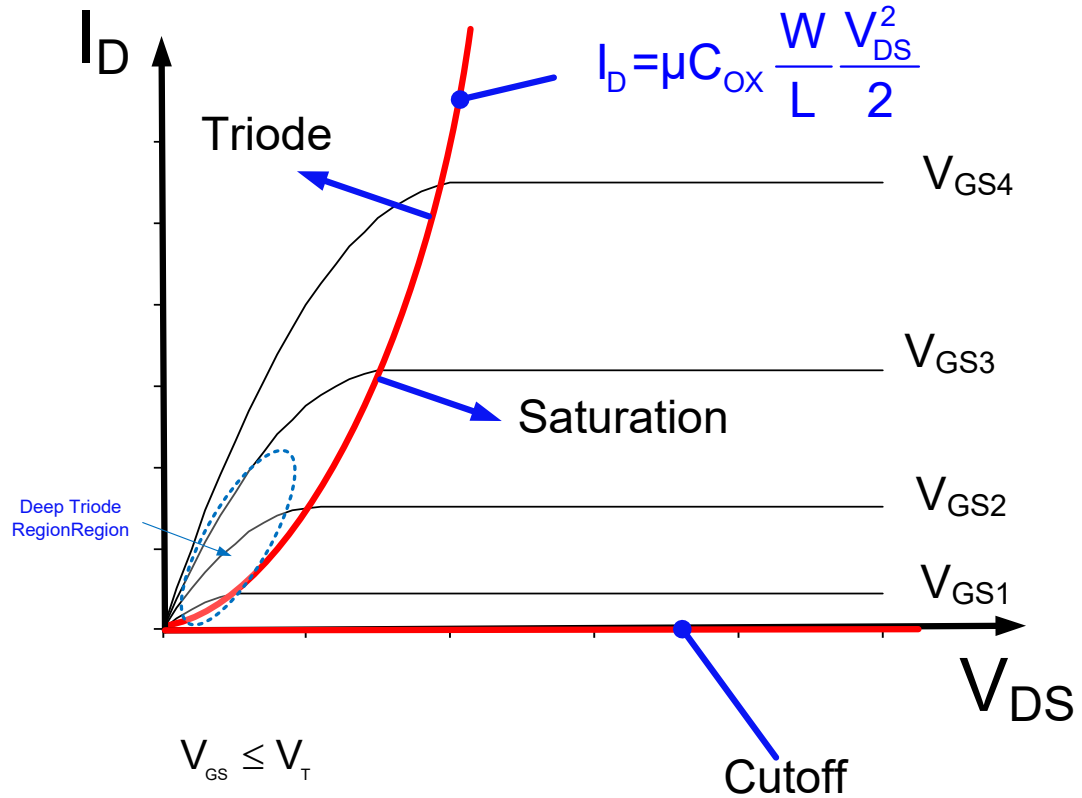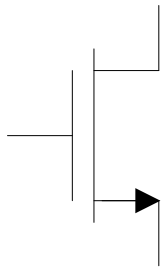
$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{OX} \dfrac{W}{L} \left(V_{GS} - V_T - \dfrac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L} \left(V_{GS} - V_T \right)^2 & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T \end{cases}$$

$$I_G = I_B = 0$$

Parabola separated triode and saturation regions and corresponds to $V_{DS} = V_{GS} - V_T$

PMOS and NMOS Models
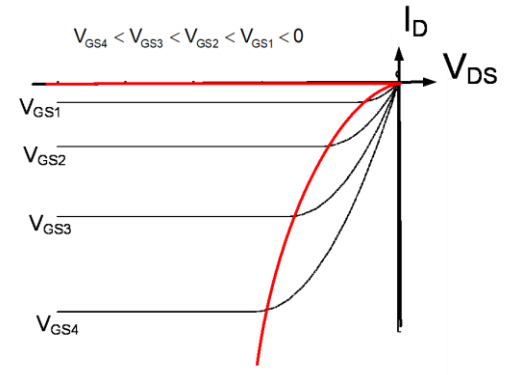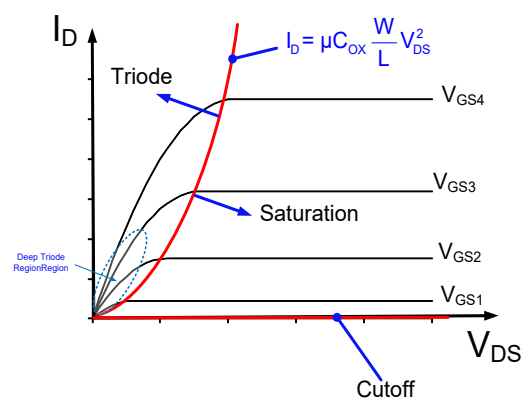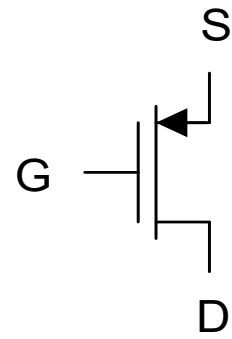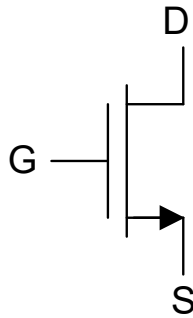
- Functional form identical, sign changes and parameter values different
- Will give details about p-channel model later

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T = 1V$ and $\mu C_{OX} = 100\mu AV^{-2}$

5V

I_D $\gtrless$ 10K

V_{OUT}

M_1

W=10u
L=2u

3V
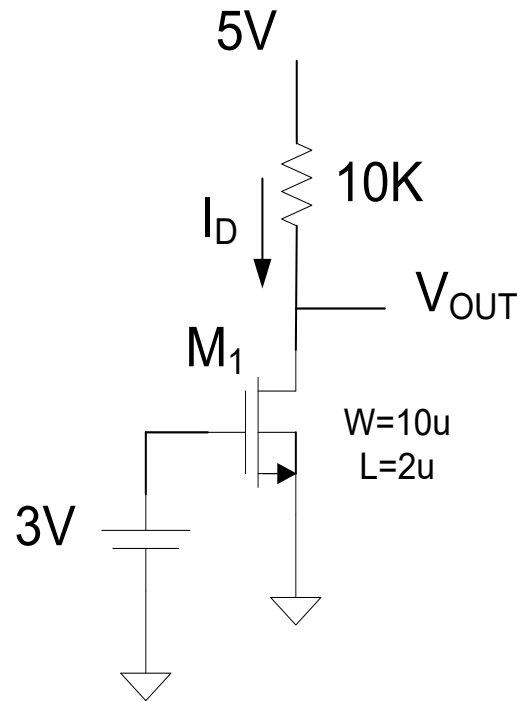
Solution:

Since $V_{GS} > V_T$, M_1 is operating in either saturation or triode region

Strategy will be to guess region of operation, solve, and then verify region

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T = 1V$ and $\mu C_{OX} = 100\mu AV^{-2}$

5V

I_D ⬇

10K

V_{OUT}

M_1

W=10u
L=2u

3V
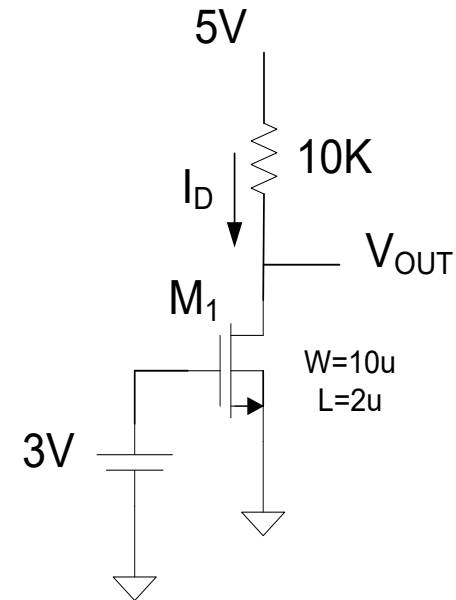
Solution:

Guess M_1 in saturation

$5V = I_D 10K + V_{OUT}$

$I_D = \dfrac{\mu C_{OX} W}{2L}(3 - V_T)^2$

Required verification: $V_{DS} > V_{GS} - V_T$

Can eliminate I_D between these 2 equations to obtain V_{OUT}

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T = 1V$ and $\mu C_{OX} = 100\mu AV^{-2}$

Guess M_1 in saturation

Required verification: $V_{DS} > V_{GS} - V_T$

$$5V = I_D 10K + V_{OUT}$$

$$I_D = \frac{\mu C_{OX} W}{2L}(3-V_T)^2$$

$$V_{OUT} = 5V - 10K\left[\frac{100\mu AV^{-2}10\mu}{2\cdot 2\mu}(2V)^2\right]$$

$$V_{OUT} = -5V$$

Verification: $V_{DS} = V_{OUT}$
-5 >? 2V - - 0 No!   So verification fails and Guess of region is invalid
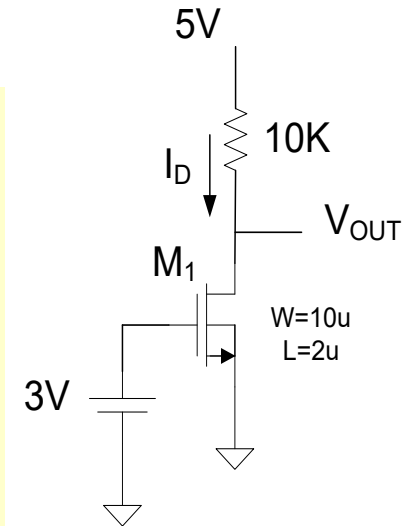
5V

I_D 10K

V_{OUT}

M_1

W=10u
L=2u

3V

Example: Determine the output voltage for the following circuit using the square-law model of the MOSFET. Assume $V_T=1V$ and $\mu C_{OX}=100\mu AV^{-2}$

Guess M_1 in triode

Required verification: $V_{DS} < V_{GS} - V_T$

$$5V = I_D 10K + V_{OUT}$$

$$I_D = \frac{\mu C_{OX} W}{L}\left(3 - V_T - \frac{V_{DS}}{2}\right)V_{DS}$$

$$V_{OUT} = 5V - 10K\left[\frac{100\mu AV^{-2}10\mu}{2\mu}\left(2V - \frac{V_{OUT}}{2}\right)V_{OUT}\right]$$

$$V_{OUT} = 5V - \left[5\left(2V - \frac{V_{OUT}}{2}\right)V_{OUT}\right]$$

Solving for V_{OUT}, obtain

$$V_{OUT} = 0.515V$$

Verification: $V_{DS} = V_{OUT}$
0.515 <? 2V   Yes!   So verification succeeds and triode region is valid
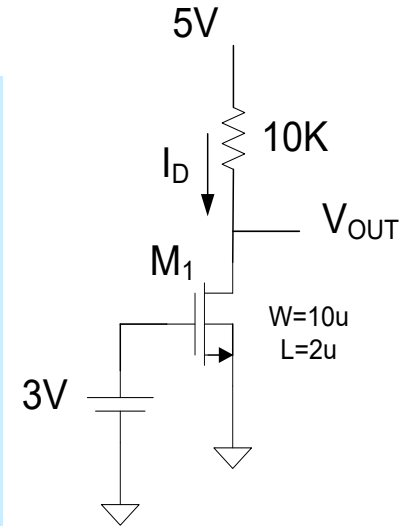
$$V_{OUT} = 0.515V$$
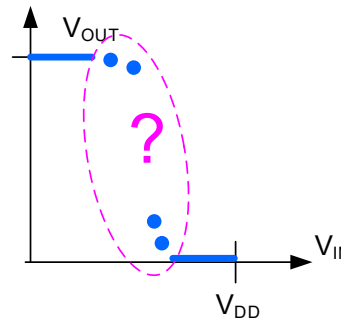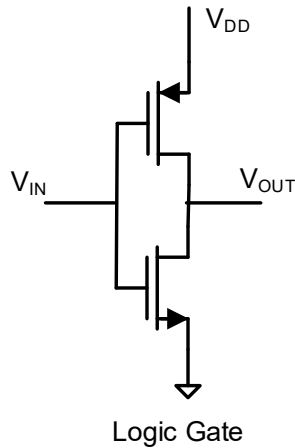
5V

10K

I_D

V_{OUT}

M_1

W=10u
L=2u

3V

Limitations of Existing Models

Logic Gate

Switch-Level Models

Simple square-law Model

Voltage Amplifier
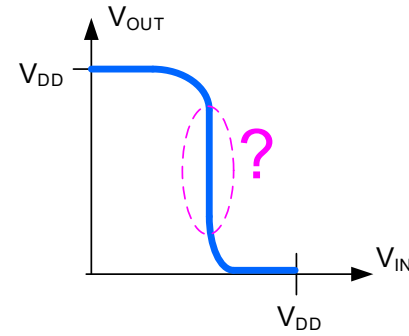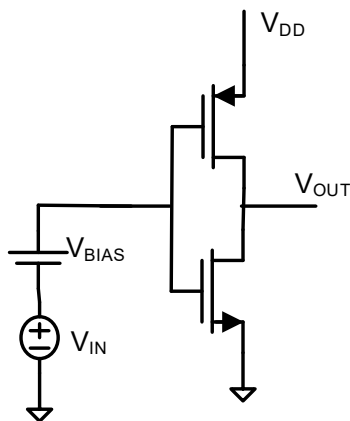
Switch-Level Models

Simple square-law Model

}

Voltage Gain
Input/Output Relationship **?**

Model Extensions

$$V_A = -\frac{1}{\lambda}$$
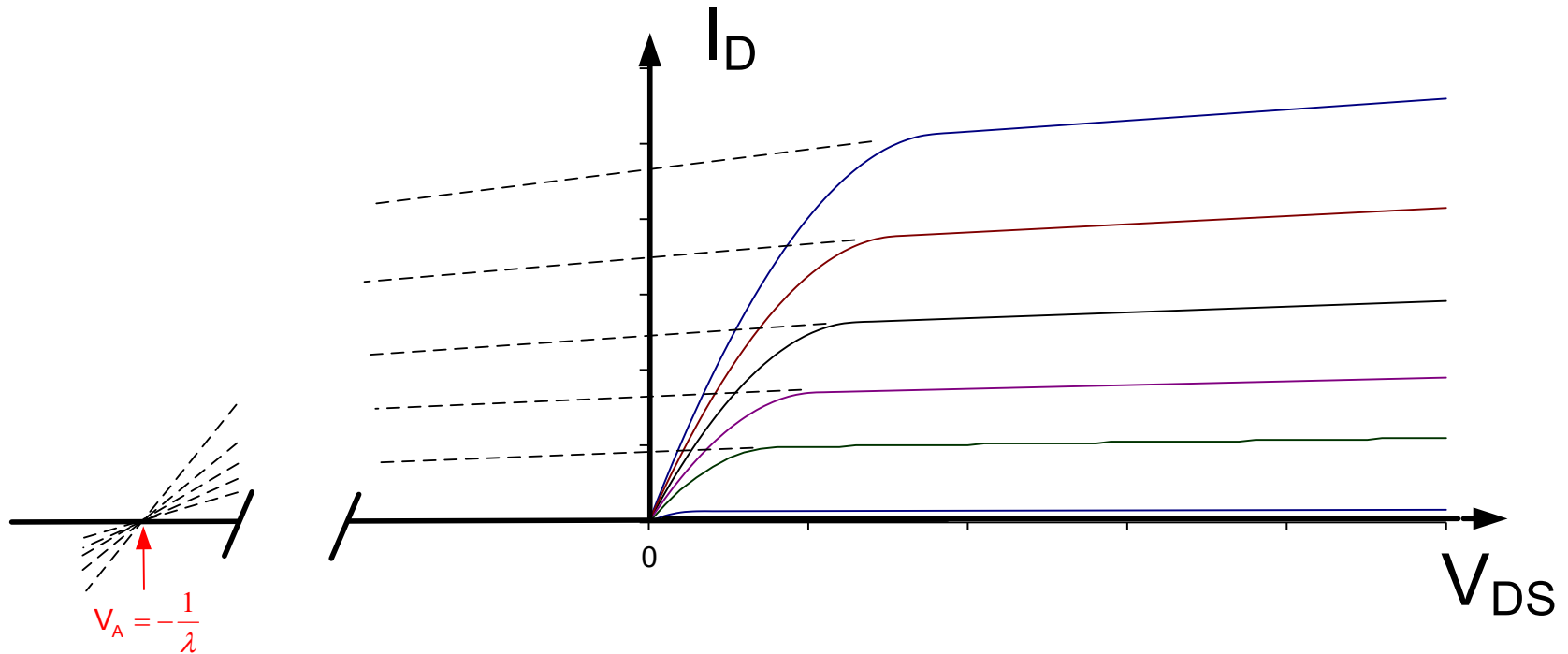
Projections intersect $-V_{DS}$ axis at same point, termed Early Voltage

Typical values from -20V to -200V

Usually use parameter λ instead of V_A in MOS model

Model Extensions
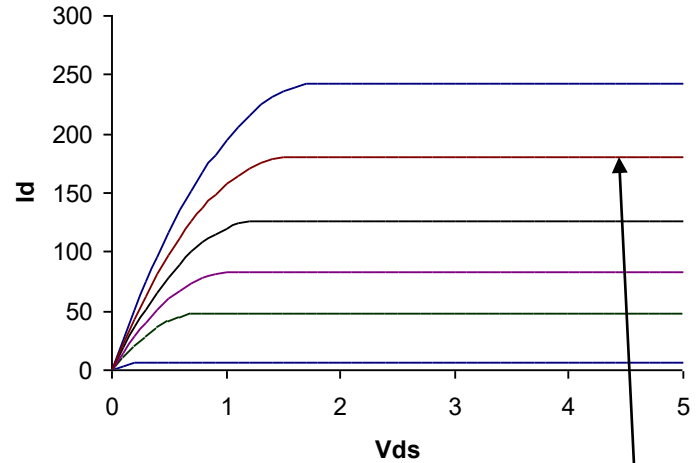
Existing Model

Slope is not 0
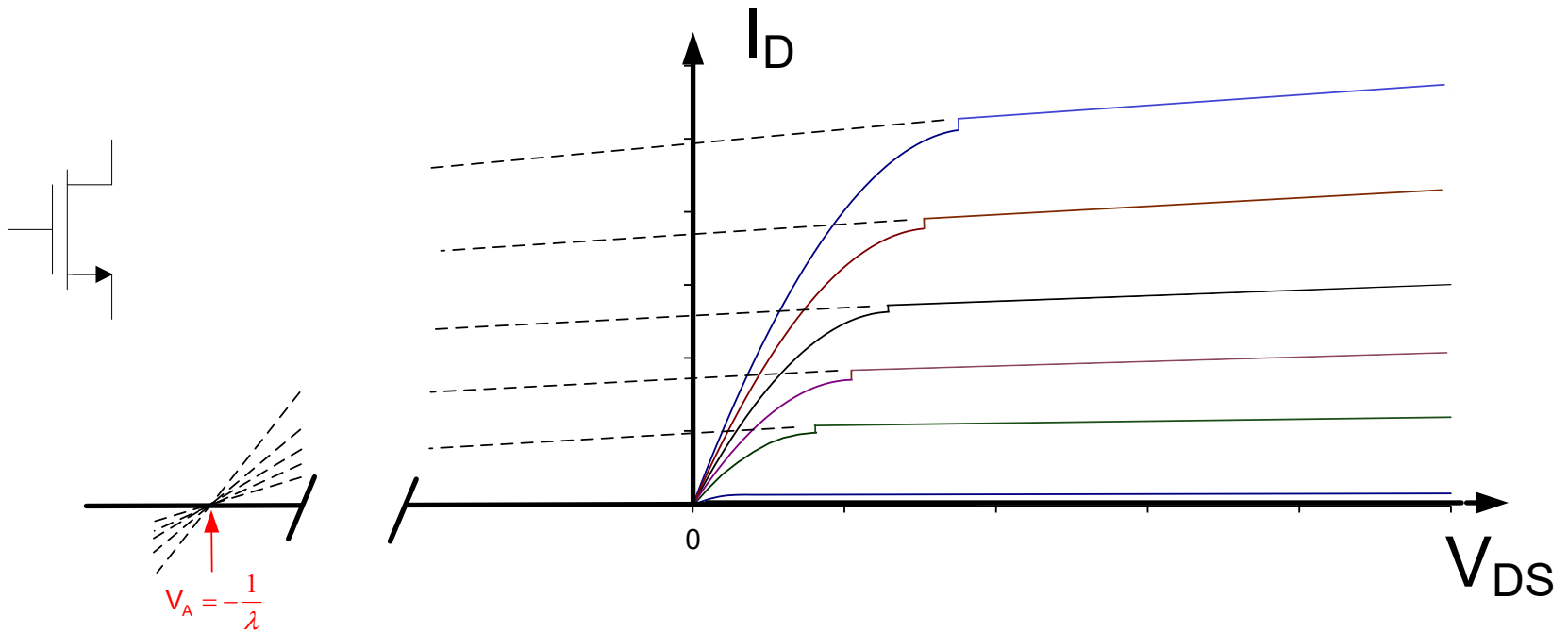
Actual Device

Model Extensions

$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T \end{cases}$$

Note: This introduces small discontinuity in model at SAT/Triode transition

Further Model Extensions

Existing model does not depend upon the bulk voltage !

Observe that changing the bulk voltage will change the electric field in the channel region !
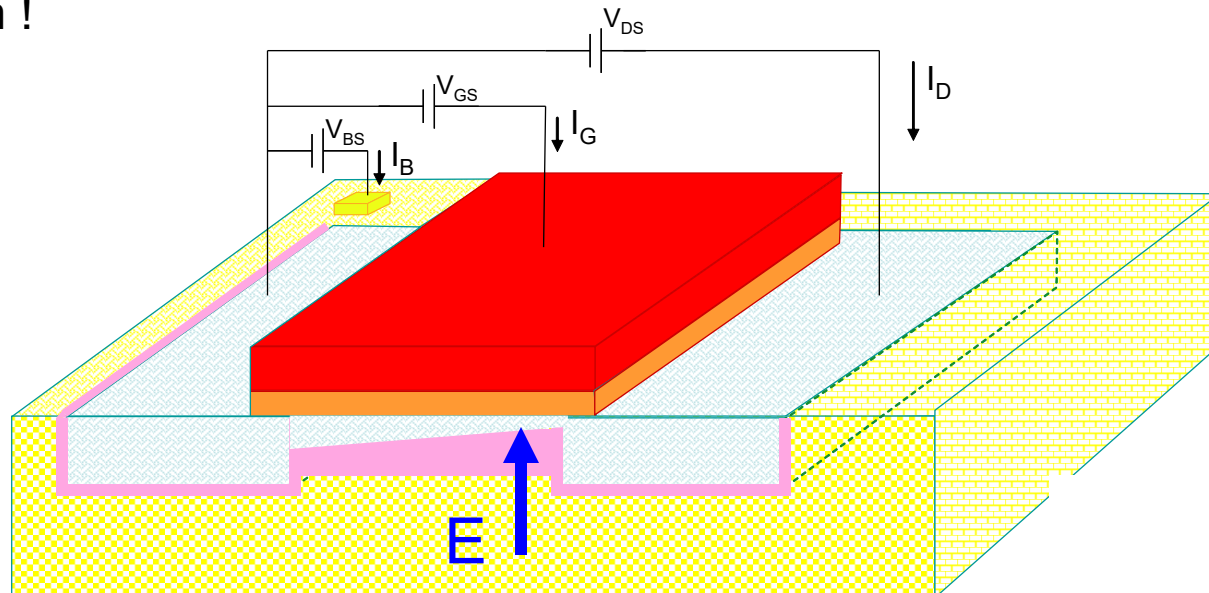
Further Model Extensions

Existing model does not depend upon the bulk voltage !

Observe that changing the bulk voltage will change the electric field in the channel region !
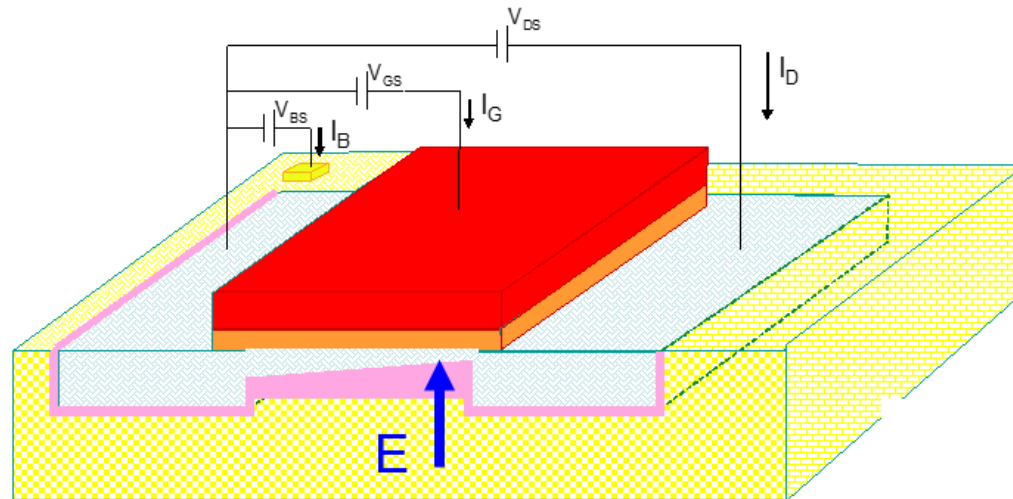
Changing the bulk voltage will change the thickness of the inversion layer

Changing the bulk voltage will change the threshold voltage of the device

$$V_T = V_{T0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

Typical Bulk Effects on Threshold Voltage for n-channel Devices

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$
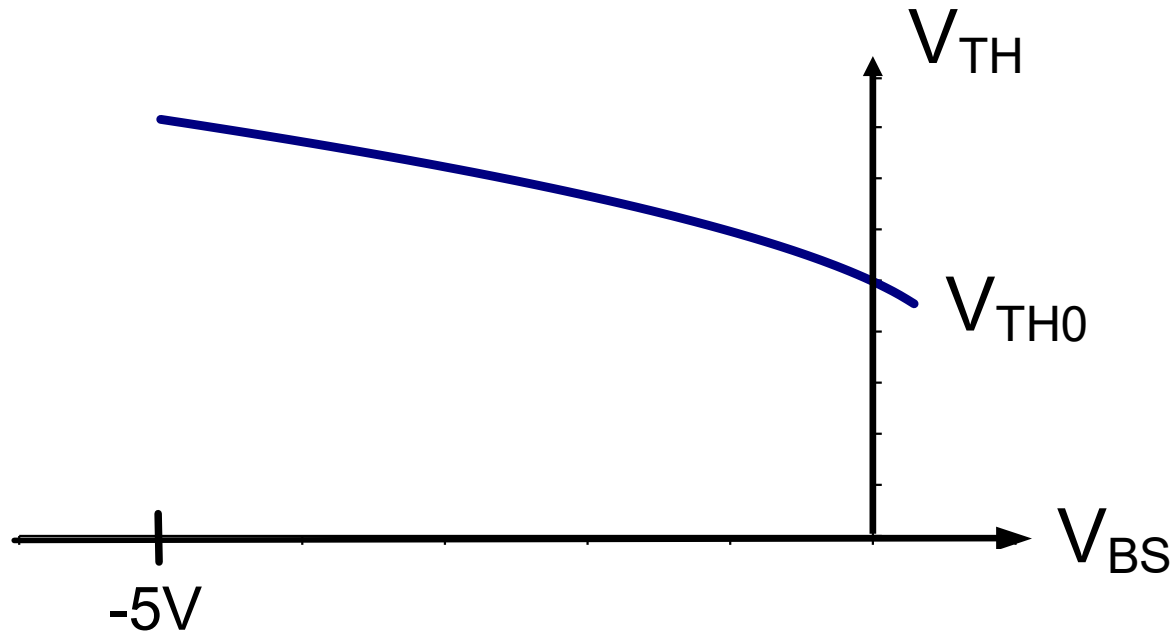
$$\gamma \cong 0.4 V^{1/2} \qquad \phi \cong 0.6 V$$

- Bulk-Diffusion Generally Reverse Biased ($V_{BS}<0$ or at least $V_{BS}<0.3V$) for n-channel
- Shift in threshold voltage with bulk voltage can be substantial
- Often $V_{BS}=0$

Typical Bulk Effects on Threshold Voltage for n-channel Devices

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$
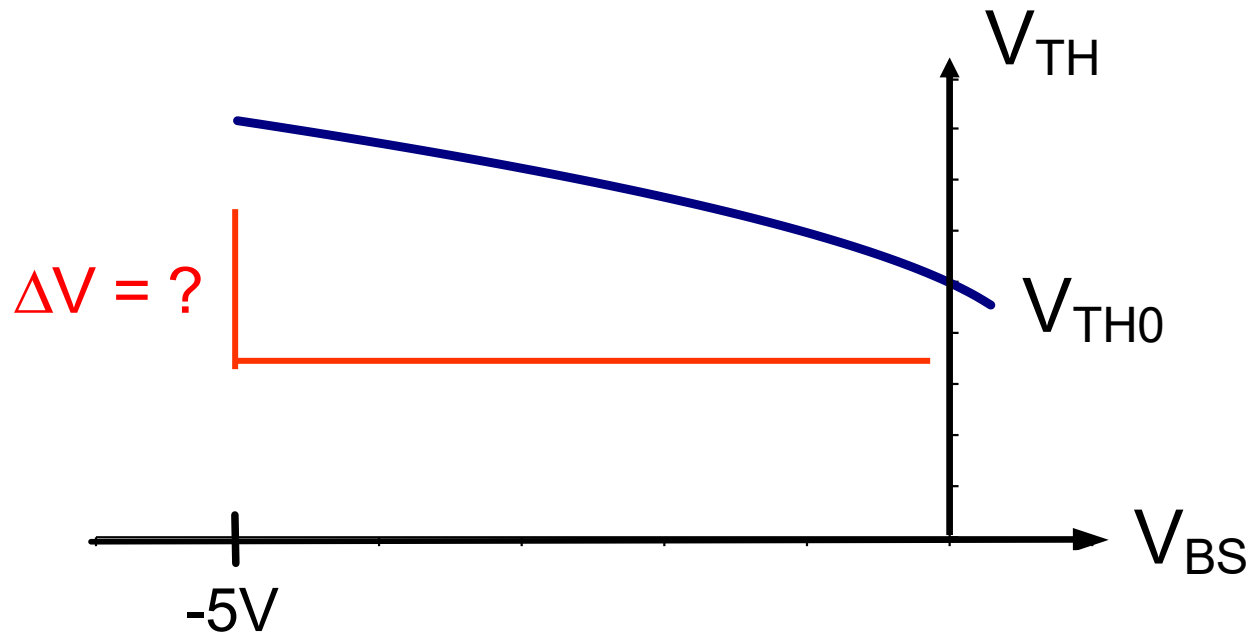
$$\gamma \cong 0.4V^{1/2} \qquad \phi \cong 0.6V$$

$$\Delta V = V_{TH} - V_{TH0} = \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$

$$\Delta V \cong 0.4 \left(\sqrt{0.6V - -5V} - \sqrt{0.6} \right) \cong 0.64V$$

Typical Bulk Effects on Threshold Voltage for p-channel Devices

$$V_{TH} = V_{TH0} - \gamma \left(\sqrt{\phi + V_{BS}} - \sqrt{\phi} \right)$$

$$\gamma \cong 0.4 V^{1/2} \qquad \phi \cong 0.6V$$
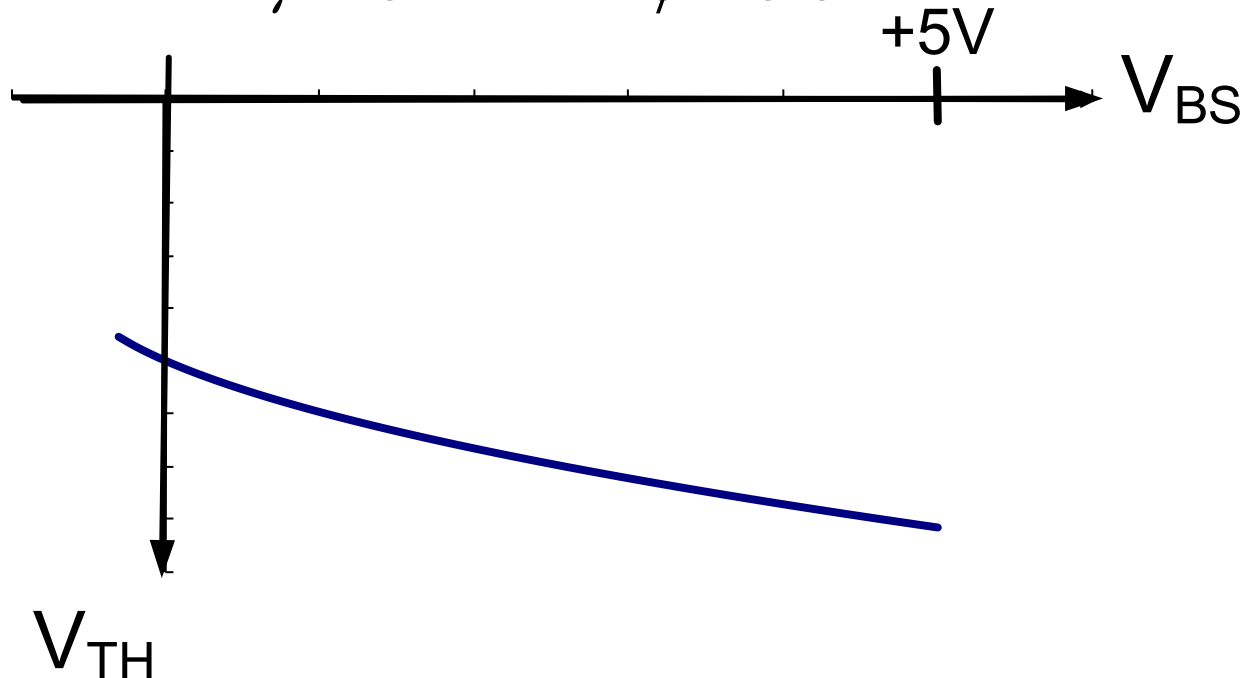
- Bulk-Diffusion Generally Reverse Biased $(V_{BS} > 0$ or at least $V_{BS} > -0.3V$ $)$ for p-channel
- Same functional form as for n-channel but $V_{TH0} < 0$
- Magnitude of threshold voltage increases with magnitude of reverse bias

Model Extension Summary

$I_G = 0$

$I_B = 0$
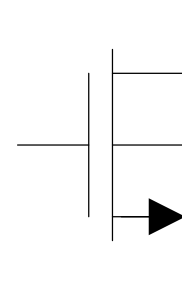
$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{T0}, \varphi, \gamma, \lambda\}$

Design Parameters : $\{W, L\}$ but only one degree of freedom W/L
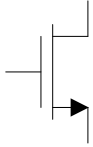
Operation Regions by Applications
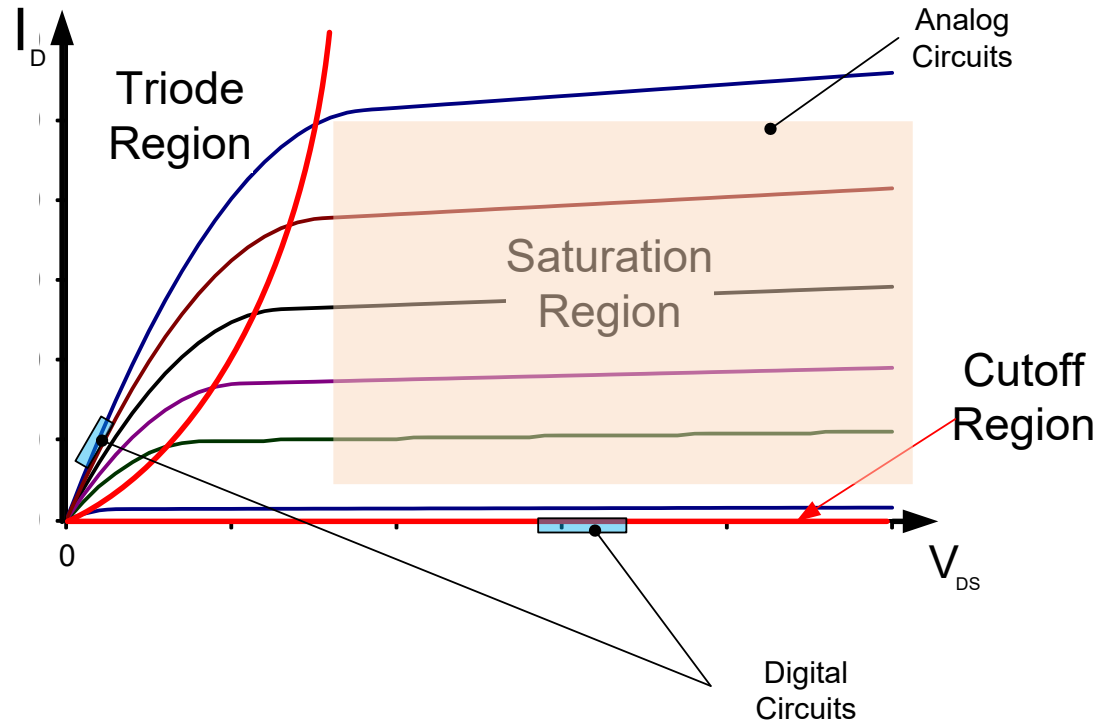
Most analog circuits operate in the saturation region

(basic VVR operates in triode and is an exception)

Most digital circuits operate in triode and cutoff regions and switch between these two with Boolean inputs

Model Extension (short devices)

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

As the channel length becomes very short, velocity saturation will occur in the channel and this will occur with electric fields around 2V/u.   So, if a gate length is around 1u, then voltages up to 2V can be applied without velocity saturation.   But, if gate length decreases and voltages are kept high, velocity saturation will occur

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \dfrac{\theta_2}{\theta_1}\mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T\right)^{\frac{\alpha}{2}} V_{DS} & V_{GS} \ge V_T \quad V_{DS} < \theta_1\left(V_{GS} - V_T\right)^{\frac{\alpha}{2}} \\ \theta_2\mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T\right)^{\alpha} & V_{GS} \ge V_T \quad V_{DS} \ge \theta_1\left(V_{GS} - V_T\right)^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, 2 ≥ α ≥ 1

Model Extension (short devices)

(n-channel device)

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\[2mm] \dfrac{\theta_2}{\theta_1}\mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_T\right)^{\frac{\alpha}{2}}V_{DS} & V_{GS}\ge V_T \quad V_{DS} < \ \theta_1\left(V_{GS}-V_T\right)^{\frac{\alpha}{2}} \\[2mm] \theta_2\mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_T\right)^{\alpha} & V_{GS}\ge V_T \quad V_{DS}\ge \ \theta_1\left(V_{GS}-V_T\right)^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, 2 ≥ α ≥ 1

No longer a square-law model (some term it an α-power or α-law model)

For long devices, α=2

Channel length modulation (λ) and bulk effects can be added to the velocity
Saturation as well

Degrading of α is not an attractive limitation of the MOSFET

Be aware of eixstance but of little use !

 (too complicated for analytical calculations, not accurate enough for simulations)

Model Extension (BSIM model)

```
.MODEL CMOSN NMOS (                                          LEVEL    = 49
+VERSION = 3.1               TNOM      = 27                  TOX      = 1.42E-8
+XJ       = 1.5E-7           NCH       = 1.7E17              VTH0     = 0.629035
+K1       = 0.8976376        K2        = -0.09255            K3       = 24.0984767
+K3B      = -8.2369696       W0        = 1.041146E-8         NLX      = 1E-9
+DVT0W    = 0                DVT1W     = 0                   DVT2W    = 0
+DVT0     = 2.7123969        DVT1      = 0.4232931           DVT2     = -0.1403765
+U0       = 451.2322004      UA        = 3.091785E-13        UB       = 1.702517E-18
+UC       = 1.22401E-11      VSAT      = 1.715884E5          A0       = 0.6580918
+AGS      = 0.130484         B0        = 2.446405E-6         B1       = 5E-6
+KETA     = -3.043349E-3     A1        = 8.18159E-7          A2       = 0.3363058
+RDSW     = 1.367055E3       PRWG      = 0.0328586           PRWB     = 0.0104806
+WR       = 1                WINT      = 2.443677E-7         LINT     = 6.999776E-8
+XL       = 1E-7             XW        = 0                   DWG      = -1.256454E-8
+DWB      = 3.676235E-8      VOFF      = -1.493503E-4        NFACTOR  = 1.0354201
+CIT      = 0                CDSC      = 2.4E-4              CDSCD    = 0
+CDSCB    = 0                ETA0      = 2.342963E-3         ETAB     = -1.5324E-4
+DSUB     = 0.0764123        PCLM      = 2.5941582           PDIBLC1  = 0.8187825
+PDIBLC2  = 2.366707E-3      PDIBLCB   = -0.0431505          DROUT    = 0.9919348
+PSCBE1   = 6.611774E8       PSCBE2    = 3.238266E-4         PVAG     = 0

+PRT      = 0                UTE       = -1.5               KT1      = -0.11
+KT1L     = 0                KT2       = 0.022               UA1      = 4.31E-9
+UB1      = -7.61E-18        UC1       = -5.6E-11            AT       = 3.3E4
+WL       = 0                WLN       = 1                   WW       = 0
+WWN      = 1                WWL       = 0                   LL       = 0
+LLN      = 1                LW        = 0                   LWN      = 1
+LWL      = 0                CAPMOD    = 2                   XPART    = 0.5
+CGDO     = 2.32E-10         CGSO      = 2.32E-10            CGBO     = 1E-9
+CJ       = 4.282017E-4      PB        = 0.9317787           MJ       = 0.4495867
+CJSW     = 3.034055E-10     PBSW      = 0.8                 MJSW     = 0.1713852
+CJSWG    = 1.64E-10         PBSWG     = 0.8                 MJSWG    = 0.1713852
+CF       = 0                PVTH0     = 0.0520855           PRDSW    = 112.8875816
+PK2      = -0.0289036       WKETA     = -0.0237483          LKETA    = 1.728324E-3   )
*
```

Model Errors with Different W/L Values
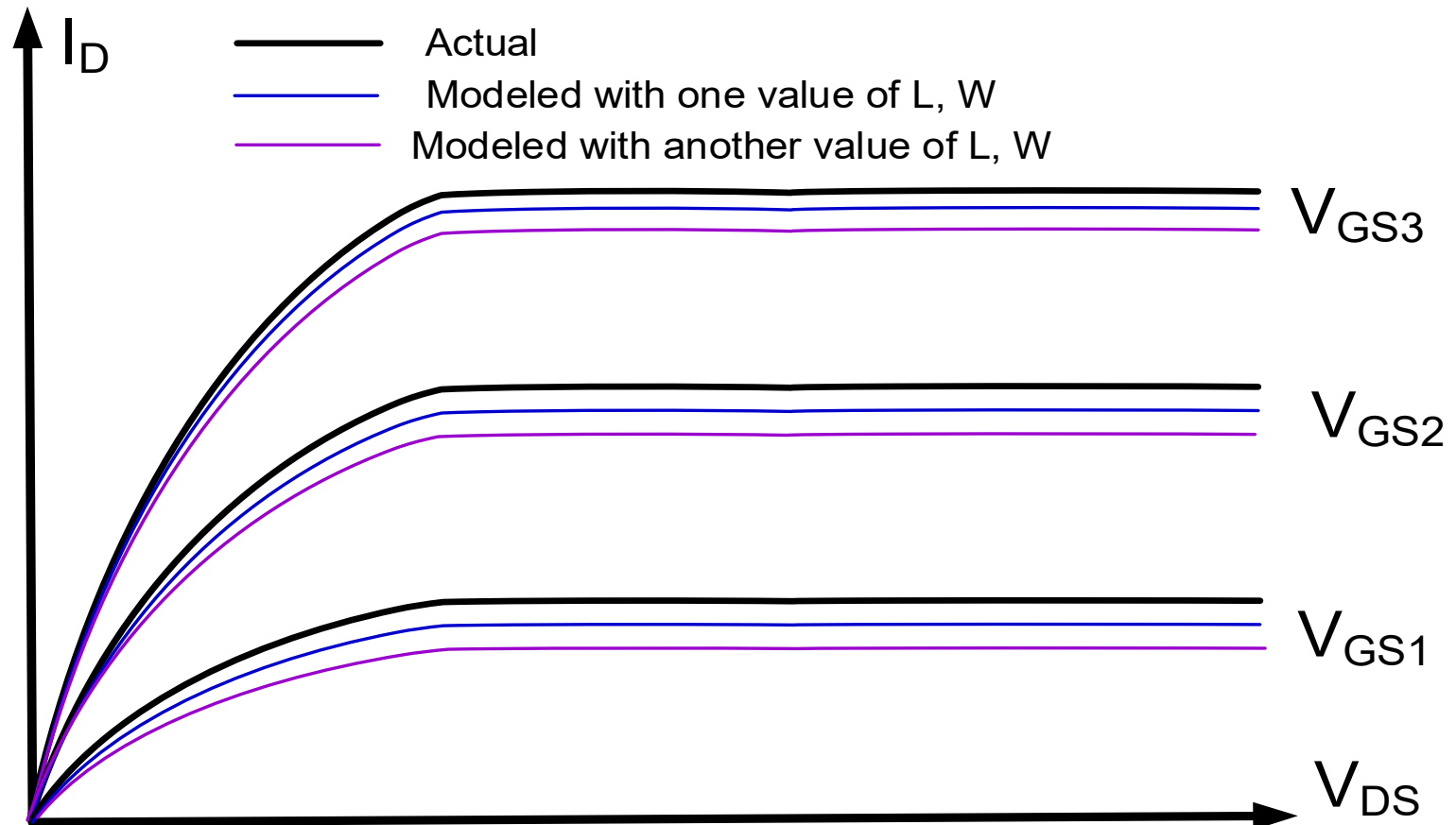
Binning models can improve model accuracy

BSIM Binning Model

- Bin on device sizes

- multiple BSIM models !

```
.MODEL CMOSN NMOS (                                          LEVEL   = 49
+VERSION = 3.1        TNOM    = 27           TOX     = 1.42E-8
+XJ      = 1.5E-7     NCH     = 1.7E17       VTH0    = 0.629035
+K1      = 0.8976376  K2      = -0.09255     K3      = 24.0984767
+K3B     = -8.2369696 W0      = 1.041146E-8  NLX     = 1E-9
+DVT0W   = 0          DVT1W   = 0            DVT2W   = 0
+DVT0    = 2.7123969  DVT1    = 0.4232931    DVT2    = -0.1403765
+U0      = 451.2322004 UA     = 3.091785E-13 UB      = 1.702517E-18
+UC      = 1.22401E-11 VSAT   = 1.715884E5   A0      = 0.6580918
+AGS     = 0.130484   B0      = 2.446405E-6  B1      = 5E-6
+KETA    = -3.043349E-3 A1    = 8.18159E-7   A2      = 0.3363058
+RDSW    = 1.367055E3 PRWG    = 0.0328586    PRWB    = 0.0104806
+WR      = 1          WINT    = 2.443677E-7  LINT    = 6.999776E-8
+XL      = 1E-7       XW      = 0            DWG     = -1.256454E-8
+DWB     = 3.676235E-8 VOFF   = -1.493503E-4 NFACTOR = 1.0354201
+CIT     = 0          CDSC    = 2.4E-4       CDSCD   = 0
+CDSCB   = 0          ETA0    = 2.342963E-3  ETAB    = -1.5324E-4
+DSUB    = 0.0764123  PCLM    = 2.5941582    PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505  DROUT   = 0.9919348
+PSCBE1  = 6.611774E8 PSCBE2  = 3.238266E-4  PVAG    = 0

+PRT     = 0          UTE     = -1.5         KT1     = -0.11
+KT1L    = 0          KT2     = 0.022        UA1     = 4.31E-9
+UB1     = -7.61E-18  UC1     = -5.6E-11     AT      = 3.3E4
+WL      = 0          WLN     = 1           WW      = 0
+WWN     = 1          WWL     = 0           LL      = 0
+LLN     = 1          LW      = 0           LWN     = 1
+LWL     = 0          CAPMOD  = 2           XPART   = 0.5
+CGDO    = 2.32E-10   CGSO    = 2.32E-10    CGBO    = 1E-9
+CJ      = 4.282017E-4 PB     = 0.9317787   MJ      = 0.4495867
+CJSW    = 3.034055E-10 PBSW  = 0.8         MJSW    = 0.1713852
+CJSWG   = 1.64E-10   PBSWG   = 0.8         MJSWG   = 0.1713852
+CF      = 0          PVTH0   = 0.0520855   PRDSW   = 112.8875816
+PK2     = -0.0289036 WKETA   = -0.0237483  LKETA   = 1.728324E-3   )
*
```

With 32 bins, this model has 3040 model parameters !

Model Changes with Process Variations

(n-ch characteristics shown)
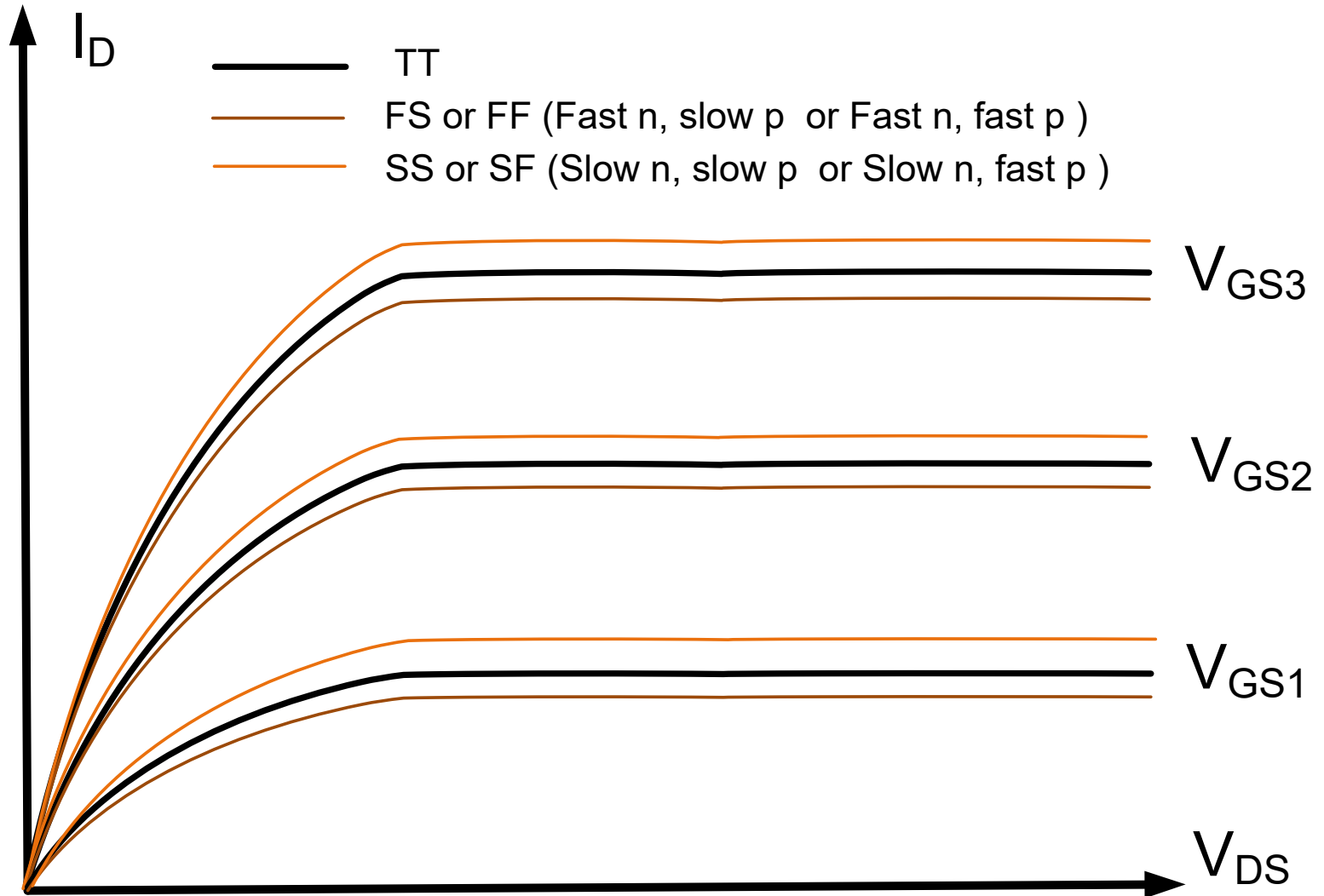
Corner models can improve model accuracy

BSIM Corner Models with Binning
- Often 4 corners in addition to nominal TT, FF, FS, SF, and SS

- bin on device sizes

```
.MODEL CMOSN NMOS (                                      LEVEL    = 49
+VERSION = 3.1          TNOM     = 27          TOX      = 1.42E-8
+XJ       = 1.5E-7      NCH      = 1.7E17      VTH0     = 0.629035
+K1       = 0.8976376   K2       = -0.09255    K3       = 24.0984767
+K3B      = -8.2369696  W0       = 1.041146E-8 NLX      = 1E-9
+DVT0W    = 0           DVT1W    = 0           DVT2W    = 0
+DVT0     = 2.7123969   DVT1     = 0.4232931   DVT2     = -0.1403765
+U0       = 451.2322004 UA       = 3.091785E-13 UB      = 1.702517E-18
+UC       = 1.22401E-11 VSAT     = 1.715884E5  A0       = 0.6580918
+AGS      = 0.130484    B0       = 2.446405E-6 B1       = 5E-6
+KETA     = -3.043349E-3 A1      = 8.18159E-7  A2       = 0.3363058
+RDSW     = 1.367055E3  PRWG     = 0.0328586   PRWB     = 0.0104806
+WR       = 1           WINT     = 2.443677E-7 LINT     = 6.999776E-8
+XL       = 1E-7        XW       = 0           DWG      = -1.256454E-8
+DWB      = 3.676235E-8 VOFF     = -1.493503E-4 NFACTOR = 1.0354201
+CIT      = 0           CDSC     = 2.4E-4      CDSCD    = 0
+CDSCB    = 0           ETA0     = 2.342963E-3 ETAB     = -1.5324E-4
+DSUB     = 0.0764123   PCLM     = 2.5941582  PDIBLC1  = 0.8187825
+PDIBLC2  = 2.366707E-3 PDIBLCB  = -0.0431505 DROUT    = 0.9919348
+PSCBE1   = 6.611774E8  PSCBE2   = 3.238266E-4 PVAG     = 0
+DELTA    = 0.01        RSH      = 83.5        MOBMOD   = 1
 +PRT      = 0           UTE      = -1.5        KT1      = -0.11
 +KT1L     = 0           KT2      = 0.022       UA1      = 4.31E-9
 +UB1      = -7.61E-18   UC1      = -5.6E-11    AT       = 3.3E4
 +WL       = 0           WLN      = 1          WW       = 0
 +WWN      = 1           WWL      = 0          LL       = 0
 +LLN      = 1           LW       = 0          LWN      = 1
 +LWL      = 0           CAPMOD   = 2          XPART    = 0.5
 +CGDO     = 2.32E-10    CGSO     = 2.32E-10   CGBO     = 1E-9
 +CJ       = 4.282017E-4 PB       = 0.9317787  MJ       = 0.4495867
 +CJSW     = 3.034055E-10 PBSW    = 0.8        MJSW     = 0.1713852
 +CJSWG    = 1.64E-10    PBSWG    = 0.8        MJSWG    = 0.1713852
 +CF       = 0           PVTH0    = 0.0520855  PRDSW    = 112.8875816
 +PK2      = -0.0289036  WKETA    = -0.0237483 LKETA    = 1.728324E-3     )
 *
```

With 32 size bins and 4 corners, this model has 15,200 model parameters !

How many models of the MOSFET do we have?

Switch-level model (2)

Square-law model

Square-law model (with λ and bulk additions)

α-law model (with λ and bulk additions)

BSIM model

BSIM model (with binning extensions)

BSIM model (with binning extensions and process corners)

The Modeling Challenge
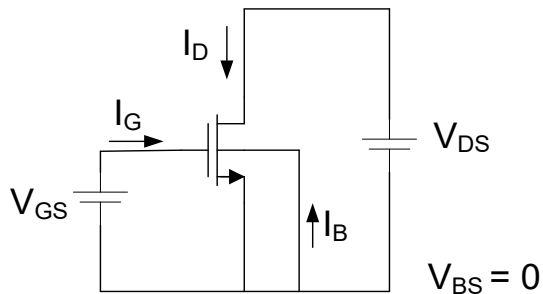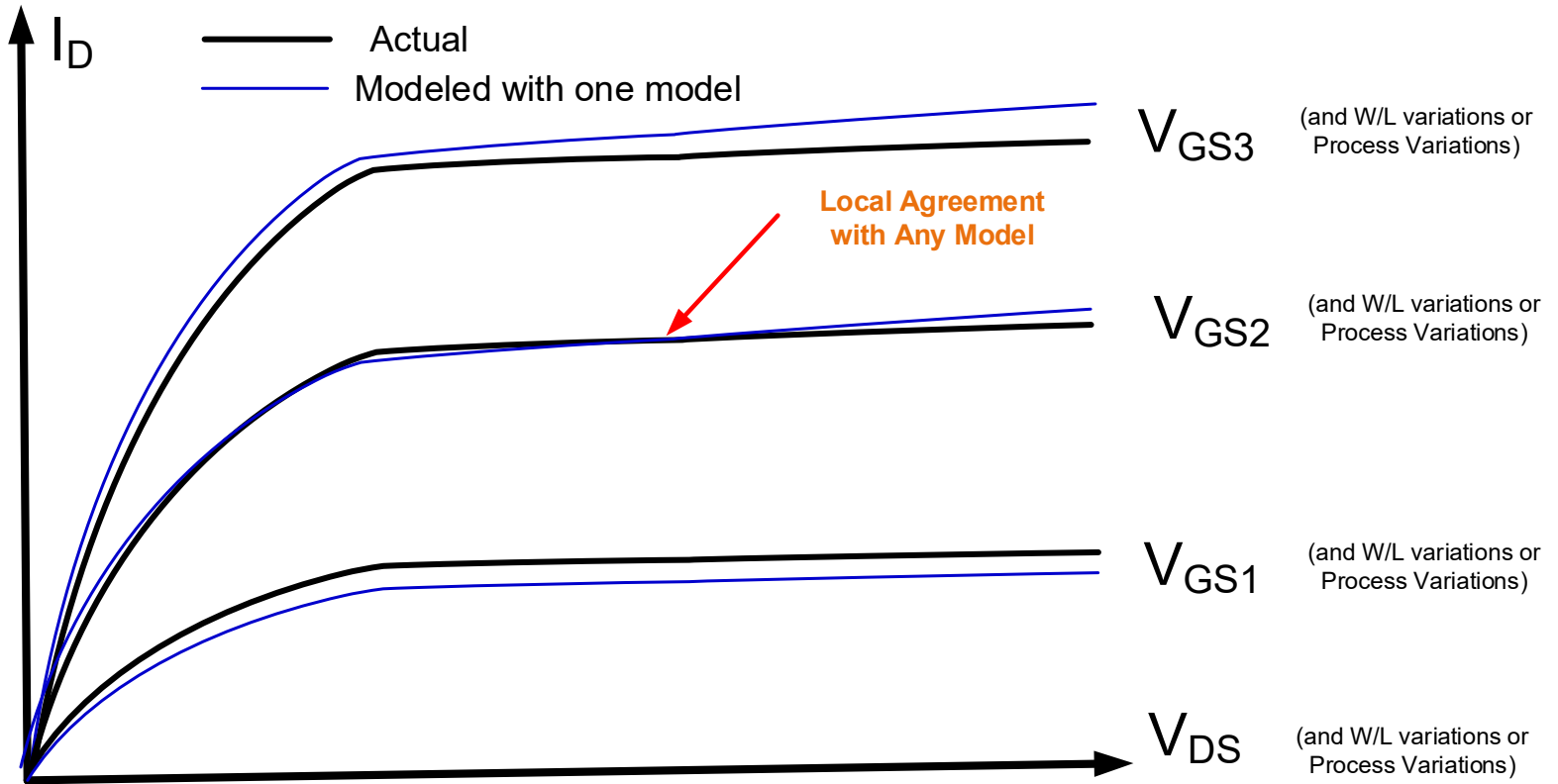
$$I_D = f_1 (V_{GS}, V_{DS})$$

$$I_G = f_2 (V_{GS}, V_{DS})$$

$$I_B = f_3 (V_{GS}, V_{DS})$$

Difficult to obtain analytical functions that accurately fit actual devices over bias, size, and process variations

Model Status

Simple dc Model → Square-Law Model

$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{ox} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{ox} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T \end{cases}$$

Small Signal

Better Analytical dc Model → Square-Law Model (with extensions for λ,γ effects)
→ Short-Channel α-law Model

Sophisticated Model for Computer Simulations → BSIM Model

Frequency Dependent Small Signal

Simpler dc Model → Switch-Level Models
• Ideal switches
• R_{SW} and C_{GS}

In the next few slides, the models we have developed will be listed and reviewed

- Square-law Model
- Switch-level Models
- Extended Square-law model
- Short-channel model
- BSIM Model
- BSIM Binning Model
- Corner Models

Square-Law Model
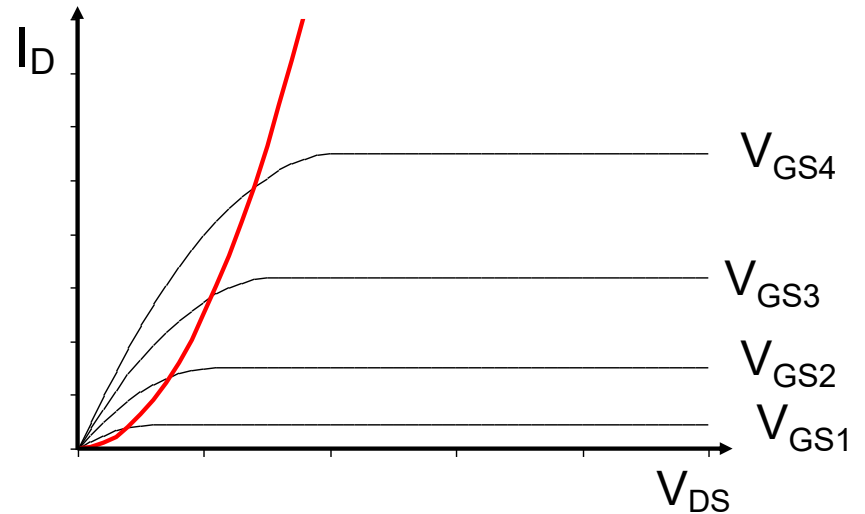
$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\[2mm] \mu C_{ox} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\[2mm] \mu C_{ox} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

Model Parameters : $\{\mu, C_{OX}, V_{T0}\}$

Design Parameters : $\{W, L\}$ but only one degree of freedom W/L

Switch-Level Models

Switch-level model including gate capacitance and drain resistance
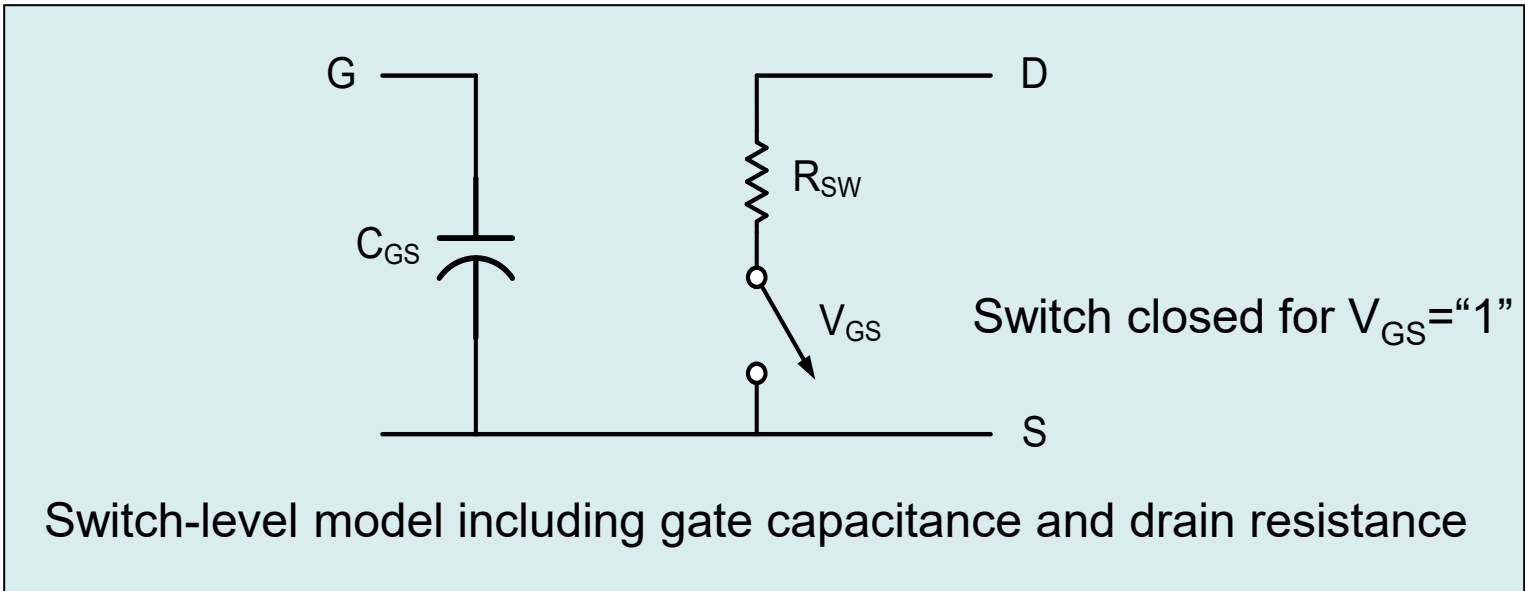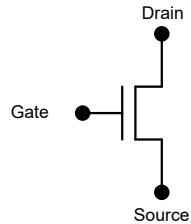
C_{GS} and R_{SW} dependent upon device sizes and process

For minimum-sized devices in a 0.5u process

$$\mathbf{C_{GS} \cong 1.5fF} \qquad \mathbf{R_{sw} \cong} \quad \begin{array}{l} \mathbf{2K\Omega \ \ n-channel} \\ \mathbf{6K\Omega \ \ p-channel} \end{array} \Big\}$$

Considerable emphasis will be placed upon device sizing to manage C_{GS} and R_{SW}

Model Parameters : $\{C_{GS}, R_{SW}\}$

Extended Square-Law Model

$I_G = 0$

$I_B = 0$
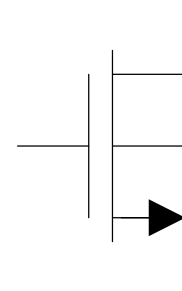
$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \geq V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right) & V_{GS} \geq V_T \quad V_{DS} \geq V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Model Parameters : $\{\mu, C_{OX}, V_{T0}, \varphi, \gamma, \lambda\}$

Design Parameters : $\{W, L\}$ but only one degree of freedom W/L

Short-Channel Model

$$I_D = \begin{cases} 0 & V_{GS} \leq V_T \\[2em] \dfrac{\theta_2}{\theta_1} \mu C_{OX} \dfrac{W}{L} \left(V_{GS} - V_T \right)^{\frac{\alpha}{2}} V_{DS} & V_{GS} \geq V_T \quad V_{DS} < \theta_1 \left(V_{GS} - V_T \right)^{\frac{\alpha}{2}} \\[2em] \theta_2 \mu C_{OX} \dfrac{W}{L} \left(V_{GS} - V_T \right)^{\alpha} & V_{GS} \geq V_T \quad V_{DS} \geq \theta_1 \left(V_{GS} - V_T \right)^{\frac{\alpha}{2}} \end{cases}$$

α is the velocity saturation index, $2 \geq \alpha \geq 1$

Channel length modulation (λ) and bulk effects can be added to the velocity
Saturation as well

BSIM model

```
.MODEL CMOSN NMOS (                                          LEVEL    = 49
+VERSION = 3.1        TNOM     = 27         TOX      = 1.42E-8
+XJ      = 1.5E-7     NCH      = 1.7E17     VTH0     = 0.629035
+K1      = 0.8976376  K2       = -0.09255   K3       = 24.0984767
+K3B     = -8.2369696 W0       = 1.041146E-8 NLX     = 1E-9
+DVT0W   = 0          DVT1W    = 0          DVT2W    = 0
+DVT0    = 2.7123969  DVT1     = 0.4232931  DVT2     = -0.1403765
+U0      = 451.2322004 UA      = 3.091785E-13 UB     = 1.702517E-18
+UC      = 1.22401E-11 VSAT    = 1.715884E5 A0       = 0.6580918
+AGS     = 0.130484   B0       = 2.446405E-6 B1      = 5E-6
+KETA    = -3.043349E-3 A1     = 8.18159E-7 A2       = 0.3363058
+RDSW    = 1.367055E3 PRWG     = 0.0328586  PRWB     = 0.0104806
+WR      = 1          WINT     = 2.443677E-7 LINT    = 6.999776E-8
+XL      = 1E-7       XW       = 0          DWG      = -1.256454E-8
+DWB     = 3.676235E-8 VOFF    = -1.493503E-4 NFACTOR = 1.0354201
+CIT     = 0          CDSC     = 2.4E-4     CDSCD    = 0
+CDSCB   = 0          ETA0     = 2.342963E-3 ETAB    = -1.5324E-4
+DSUB    = 0.0764123  PCLM     = 2.5941582  PDIBLC1 = 0.8187825
+PDIBLC2 = 2.366707E-3 PDIBLCB = -0.0431505 DROUT   = 0.9919348
+PSCBE1  = 6.611774E8 PSCBE2   = 3.238266E-4 PVAG    = 0

+PRT     = 0          UTE      = -1.5       KT1      = -0.11
+KT1L    = 0          KT2      = 0.022      UA1      = 4.31E-9
+UB1     = -7.61E-18  UC1      = -5.6E-11   AT       = 3.3E4
+WL      = 0          WLN      = 1          WW       = 0
+WWN     = 1          WWL      = 0          LL       = 0
+LLN     = 1          LW       = 0          LWN      = 1
+LWL     = 0          CAPMOD   = 2          XPART    = 0.5
+CGDO    = 2.32E-10   CGSO     = 2.32E-10   CGBO     = 1E-9
+CJ      = 4.282017E-4 PB      = 0.9317787  MJ       = 0.4495867
+CJSW    = 3.034055E-10 PBSW   = 0.8        MJSW     = 0.1713852
+CJSWG   = 1.64E-10   PBSWG    = 0.8        MJSWG    = 0.1713852
+CF      = 0          PVTH0    = 0.0520855  PRDSW    = 112.8875816
+PK2     = -0.0289036 WKETA    = -0.0237483 LKETA    = 1.728324E-3   )
*
```

Note this model has 95 model parameters !

BSIM Binning Model

- Bin on device sizes

- multiple BSIM models !

```
.MODEL CMOSN NMOS (                                          LEVEL    = 49
+VERSION = 3.1          TNOM     = 27           TOX      = 1.42E-8
+XJ      = 1.5E-7       NCH      = 1.7E17       VTH0     = 0.629035
+K1      = 0.8976376    K2       = -0.09255     K3       = 24.0984767
+K3B     = -8.2369696   W0       = 1.041146E-8  NLX      = 1E-9
+DVT0W   = 0            DVT1W    = 0            DVT2W    = 0
+DVT0    = 2.7123969    DVT1     = 0.4232931    DVT2     = -0.1403765
+U0      = 451.2322004  UA       = 3.091785E-13 UB       = 1.702517E-18
+UC      = 1.22401E-11  VSAT     = 1.715884E5   A0       = 0.6580918
+AGS     = 0.130484     B0       = 2.446405E-6  B1       = 5E-6
+KETA    = -3.043349E-3 A1       = 8.18159E-7   A2       = 0.3363058
+RDSW    = 1.367055E3   PRWG     = 0.0328586    PRWB     = 0.0104806
+WR      = 1            WINT     = 2.443677E-7  LINT     = 6.999776E-8
+XL      = 1E-7         XW       = 0            DWG      = -1.256454E-8
+DWB     = 3.676235E-8  VOFF     = -1.493503E-4 NFACTOR  = 1.0354201
+CIT     = 0            CDSC     = 2.4E-4       CDSCD    = 0
+CDSCB   = 0            ETA0     = 2.342963E-3  ETAB     = -1.5324E-4
+DSUB    = 0.0764123    PCLM     = 2.5941582    PDIBLC1  = 0.8187825
+PDIBLC2 = 2.366707E-3  PDIBLCB  = -0.0431505   DROUT    = 0.9919348
+PSCBE1  = 6.611774E8   PSCBE2   = 3.238266E-4  PVAG     = 0

+PRT     = 0            UTE      = -1.5         KT1      = -0.11
+KT1L    = 0            KT2      = 0.022        UA1      = 4.31E-9
+UB1     = -7.61E-18    UC1      = -5.6E-11     AT       = 3.3E4
+WL      = 0            WLN      = 1            WW       = 0
+WWN     = 1            WWL      = 0            LL       = 0
+LLN     = 1            LW       = 0            LWN      = 1
+LWL     = 0            CAPMOD   = 2            XPART    = 0.5
+CGDO    = 2.32E-10     CGSO     = 2.32E-10     CGBO     = 1E-9
+CJ      = 4.282017E-4  PB       = 0.9317787    MJ       = 0.4495867
+CJSW    = 3.034055E-10 PBSW     = 0.8          MJSW     = 0.1713852
+CJSWG   = 1.64E-10     PBSWG    = 0.8          MJSWG    = 0.1713852
+CF      = 0            PVTH0    = 0.0520855    PRDSW    = 112.8875816
+PK2     = -0.0289036   WKETA    = -0.0237483   LKETA    = 1.728324E-3    )
*
```

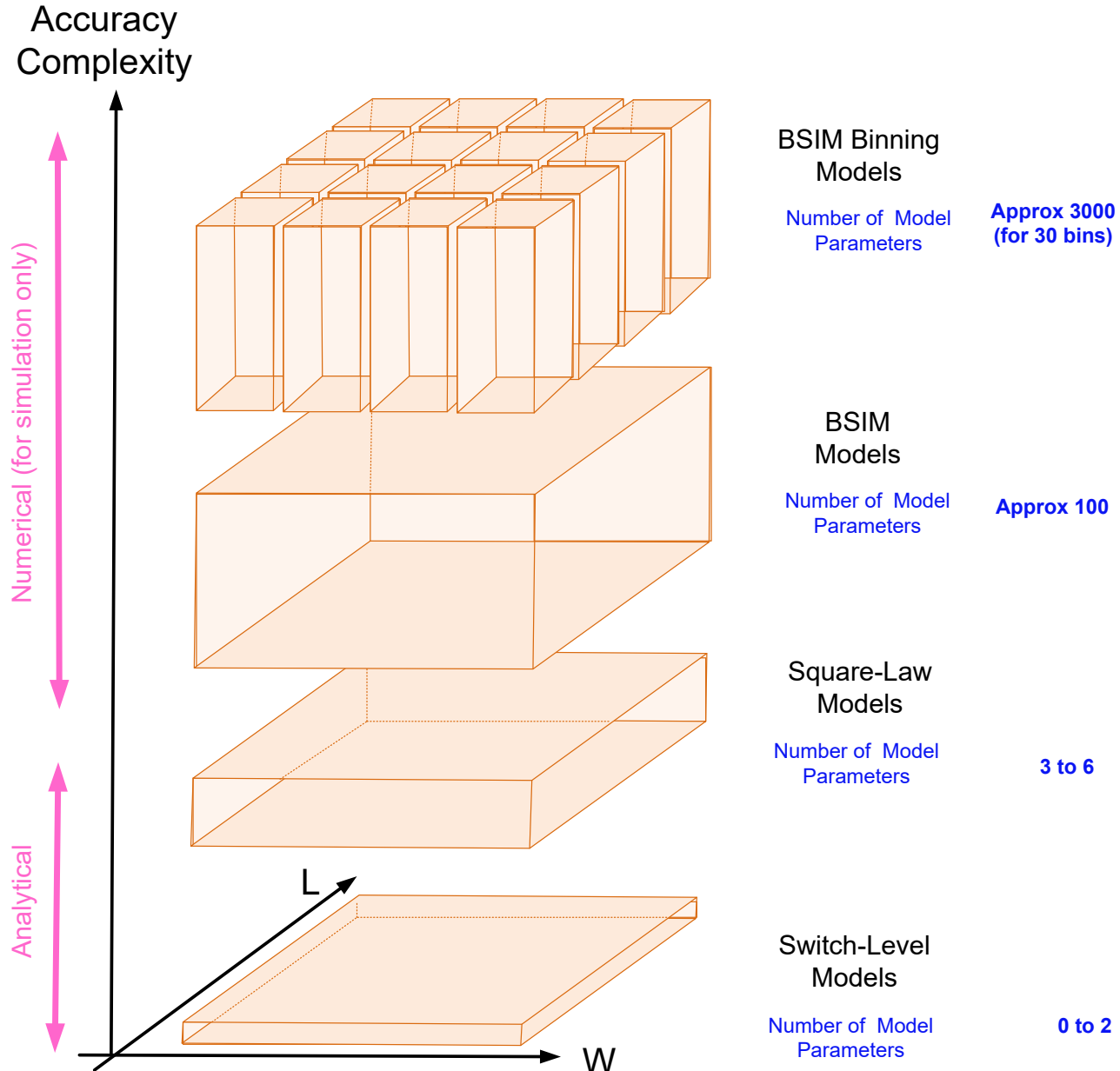With 32 bins, this model has 3040 model parameters !

BSIM Corner Models

- Often 4 corners in addition to nominal TT, FF, FS, SF, and SS

- five different BSIM models !

```
.MODEL CMOSN NMOS (                              LEVEL    = 49
+VERSION = 3.1        TNOM     = 27              TOX      = 1.42E-8
+XJ       = 1.5E-7    NCH      = 1.7E17          VTH0     = 0.629035
+K1       = 0.8976376 K2       = -0.09255        K3       = 24.0984767
+K3B      = -8.2369696 W0      = 1.041146E-8     NLX      = 1E-9
+DVT0W    = 0         DVT1W    = 0               DVT2W    = 0
+DVT0     = 2.7123969 DVT1     = 0.4232931       DVT2     = -0.1403765
+U0       = 451.2322004 UA     = 3.091785E-13    UB       = 1.702517E-18
+UC       = 1.22401E-11 VSAT   = 1.715884E5      A0       = 0.6580918
+AGS      = 0.130484  B0       = 2.446405E-6     B1       = 5E-6
+KETA     = -3.043349E-3 A1    = 8.18159E-7      A2       = 0.3363058
+RDSW     = 1.367055E3 PRWG    = 0.0328586       PRWB     = 0.0104806
+WR       = 1         WINT     = 2.443677E-7     LINT     = 6.999776E-8
+XL       = 1E-7      XW       = 0               DWG      = -1.256454E-8
+DWB      = 3.676235E-8 VOFF   = -1.493503E-4    NFACTOR  = 1.0354201
+CIT      = 0         CDSC     = 2.4E-4          CDSCD    = 0
+CDSCB    = 0         ETA0     = 2.342963E-3     ETAB     = -1.5324E-4
+DSUB     = 0.0764123 PCLM    = 2.5941582        PDIBLC1  = 0.8187825
+PDIBLC2  = 2.366707E-3 PDIBLCB = -0.0431505     DROUT    = 0.9919348
+PSCBE1   = 6.611774E8 PSCBE2  = 3.238266E-4     PVAG     = 0
+DELTA    = 0.01      RSH      = 83.5            MOBMOD   = 1
 +PRT     = 0         UTE      = -1.5            KT1      = -0.11
 +KT1L    = 0         KT2      = 0.022           UA1      = 4.31E-9
 +UB1     = -7.61E-18 UC1      = -5.6E-11        AT       = 3.3E4
 +WL      = 0         WLN      = 1               WW       = 0
 +WWN     = 1         WWL      = 0               LL       = 0
 +LLN     = 1         LW       = 0               LWN      = 1
 +LWL     = 0         CAPMOD   = 2               XPART    = 0.5
 +CGDO    = 2.32E-10  CGSO     = 2.32E-10        CGBO     = 1E-9
 +CJ      = 4.282017E-4 PB     = 0.9317787       MJ       = 0.4495867
 +CJSW    = 3.034055E-10 PBSW  = 0.8             MJSW     = 0.1713852
 +CJSWG   = 1.64E-10  PBSWG    = 0.8             MJSWG    = 0.1713852
 +CF      = 0         PVTH0    = 0.0520855       PRDSW    = 112.8875816
 +PK2     = -0.0289036 WKETA   = -0.0237483      LKETA    = 1.728324E-3    )
 *
```
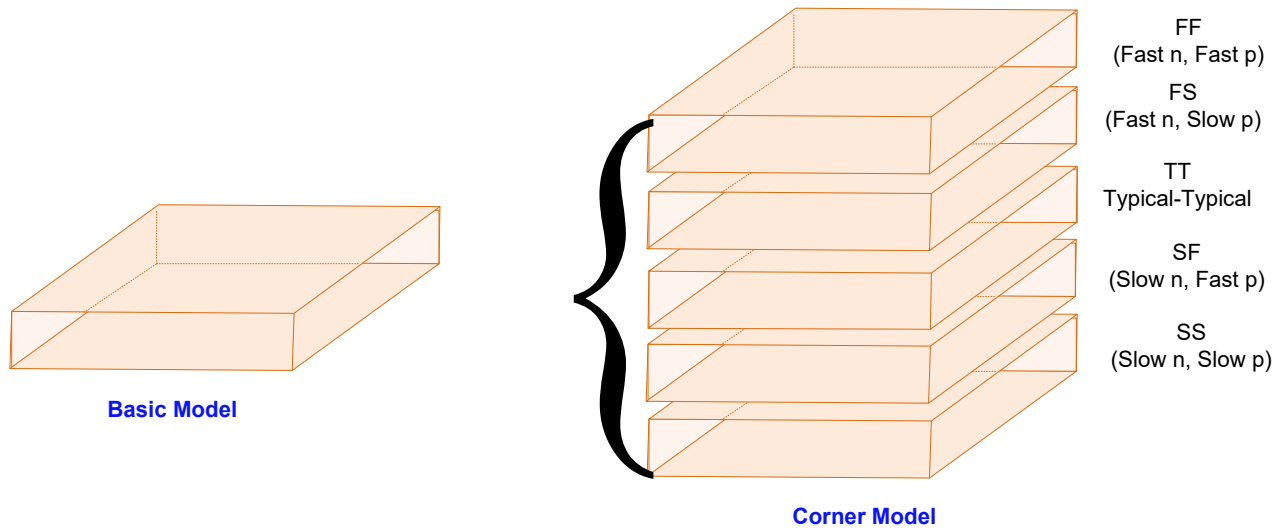
TT: typical-typical
FF: fast n, fast p
FS: fast n, slow p
SF: slow n, fast p
SS: slow n, slow p

With 4 corners, this model has 475 model parameters !

Hierarchical Model Comparisons

Accuracy
Complexity

Numerical (for simulation only)

Analytical

L

W

BSIM Binning
Models

Number of Model
Parameters — **Approx 3000
(for 30 bins)**

BSIM
Models

Number of Model
Parameters — **Approx 100**

Square-Law
Models

Number of Model
Parameters — **3 to 6**

Switch-Level
Models

Number of Model
Parameters — **0 to 2**

Corner Models

FF
(Fast n, Fast p)

FS
(Fast n, Slow p)

TT
Typical-Typical

SF
(Slow n, Fast p)

SS
(Slow n, Slow p)

Basic Model

Corner Model

Applicable at any level in model hierarchy (same model, different parameters)

Often 4 corners (FF, FS, SF, SS) used but sometimes many more

Designers must provide enough robustness so good yield at all corners

Stay Safe and Stay Healthy !

End of Lecture 17